Understanding "reform" at the collegiate level:

Exploring students' experiences in reform Calculus

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Paper presented at the RUME (Research in Undergraduate Mathematics Education) conference, Chicago, September, 2000.

Acknowledgements: The project is supported by a grant from the National Science Foundation. We would like to acknowledge the contributions of the following University of Michigan undergraduate students who worked as Research Assistants on this project during the 1999-2000 year: Alissa Belkin, Kelly Maltese, Brian My, Sharon Risch, Brian Walby, William Nham, Matthew Dilliard, Adrienne Frogner-Howell, and Abby Magid. We greatly appreciate the assistance and cooperation of Pat Shure and the rest of the faculty, graduate students, and staff of the Mathematics Department of the University of Michigan in our research.

In the decade since the 1989 publication of NCTM's *Curriculum and Evaluation Standards,* curricula consistent with that vision for pre-college mathematics education have been written and implemented in many U.S. elementary and secondary schools. A related but distinct movement at the university level has led to the development and use of Standards-based Calculus curricula. Evaluation and assessment studies are beginning to focus on how these curricula perform relative to traditional curricula in supporting student learning and positive attitudes (e. g., Hoover, Zawojewski, & Ridgway, 1997; Schoen, Hirsch, & Ziebarth, 1998). Educators have also begun to examine the processes of teacher learning and change in implementing reforms (e.g., Lloyd & Wilson, 1998; Putnam, Heaton, Prawat, & Remilliard, 1992). But the implementation of these reform curricula has been often "spotty," nationally and regionally. Many school districts and universities have chosen Standards-based curricular materials for one or more levels of their K–16 system while retaining older curricular materials that reflect less of the Standards vision at other levels. Often, such non-systemic implementations reflect substantial differences, within communities and between school buildings, on how mathematics is best taught and learned.

These "spotty" implementations can create conditions where students experience very different expectations for what it means to think, know, and do mathematics. For example, one curriculum may value and reward students' ability to explain their thinking, work productively with other students, and undertake large-scale inquiry relatively independently, while the previous (or subsequent) program of curriculum and teaching may not. As students move between schools (and sometimes even within schools), such transitions between Standards-based and more traditional curricula are increasingly common. Yet very little attention has been paid to studying the effects of these potential shifts for students (cf. Smith, Star, Herbel-Eisenmann, & Jansen, 2000; Walker, 1999). As students move into and out of mathematics classrooms and curricula which reflect different expectations and ways of knowing, what do they notice? How are their learning and attitudes toward mathematics affected? How do they adjust to changes when they recognize them?

These are the sort of questions we address in the research presented in this paper. We have just completed the first year (1999-2000) of a three-year, NSF-funded project which examines students' mathematical transitions at four sites (2 high schools and 2 universities). At each site, students move between programs with “traditional” expectations for mathematical work and those with expectations more consonant with the NCTM Standards (in short, "reform" curricula). At two of the sites (one high school and one university), students move from a "traditional" curriculum to a "reform" curriculum; the reverse (a move from "reform" to "traditional") is true at the other two sites. The specific "reform"-oriented programs included in our study are the Connected Mathematics Project [CMP] (Lappan, et al., 1995), Core Plus Mathematics Project [CPMP] (Hirsch, Coxford, Fey, & Schoen, 1996), and the Harvard Consortium Calculus program [Harvard Calculus] (Hughes-Hallett, Gleason, et al., 1994). This paper focuses exclusively on one of the university sites -- the University of Michigan [U-M], where many students move from traditional high school programs to courses taught using the Harvard Consortium materials.

Perspective. The concept of "mathematical transitions" has not been examined in academic research, so we want to be clear about our basic terms and meanings. We call marked differences between students' prior notions of what it means to think and act mathematically and how they are expected to think and act in their current classroom "mathematical discontinuities". These discontinuities "happen" in and around some mathematical content, but they refer to students' experience of new (or different) expectations for their activity with that content—not to the content alone. Mathematical transitions are students' conscious experience of and responses to those discontinuities: how they experience and understand the difference(s), how they respond (or not) to them, and how they understand the results of these responses. Mathematical discontinuities and transitions are naturally occurring phenomena, but they become more likely in periods of curricular reform where "implementation" is uneven.

Research questions. Our work centers around the following research questions. First, what do students notice as different in their current mathematics experience, as compared to what was experienced one year ago? Answering this initial question requires that we characterize and compare the intended (text materials) and enacted (teaching practices) curriculum in students' current mathematics classes with the corresponding curricula in their high school programs. Second, what mathematical transitions do students experience? Third, for those students who do experience transitions, what strategies and resources do they try out in an attempt to adjust to the discontinuities? What strategies and resources do they ignore? What are the consequences of trying out those strategies and resources?

Mathematical discontinuities at the University of Michigan. The University of Michigan is an ideal location to search for students' experiences of different mathematical expectations. Most U-M first-year students have taken 4 years of mathematics in high school and have used "traditional" mathematics curricula in all courses, up to and often including an AP Calculus course. Upon entering U-M, all students who choose to take math (other than those in honors courses) take courses which use a reform curriculum -- the Harvard Consortium materials. At U-M, the Harvard Consortium curricula are used in three, semester-long[1] courses: PreCalculus [Math 105] (Connally et al., 1998) and Calculus I [Math 115] and II [Math 116] (Hughes-Hallett et al., 1994). The Harvard Consortium materials claim to differ from more traditional curricula along several dimensions, including an emphasis on real-world and contextual problems, a greater focus on multiple representations of topics (geometric, numerical, analytical, and verbal -- “The Rule of Four”), the development of formal definitions and procedures from work on practical problems (“The Way of Archimedes”), and an increased depth of understanding rather than breadth of coverage.

All three U-M courses are taught in many small sections of approximately 25-30 students. Although each section of each course is taught independently by a single instructor, all sections within a course share common homework assignments, common unit tests, and a common final exam. Group work is required in and out of class. In class, students typically sit at tables in groups of 4 and are often encouraged to work on problems with those sitting around them. Out of class, students are assigned a group and

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[1] In this paper, we use "semester" and "term" interchangeably. We call the term which runs from September to December the "first" semester or term and the one that runs from January to May the "second" semester or term.

are given a homework assignment every week to be done in groups. This group homework assignment is typically composed of problems which are more difficult than those the student might see in class or on tests, ostensibly to encourage more of a group effort.

After the Calc II (Math 116), students who continue taking mathematics move on to an introductory course on Differential Equations (Math 215). Math 215 and all subsequent courses following Math 116 do not use the Harvard or other reform-oriented materials.

We begin by describing our data collection method and the students who chose to participate in our study. We then characterize these students' experience in the U-M Calculus program, in terms of their achievement and also their perceptions of differences between high school and college math.

Method

Participants. Nineteen first-year students at the University of Michigan volunteered for this study (10 females; 9 males). Students were recruited by posting flyers in the building where their mathematics classes met; students were compensated $250 per semester for their participation in this study. In order to eligible to participate, students had to be enrolled full-time as first-year students at U-M, over the age of 18, and have attended high school in the state of Michigan. All students who volunteered and met with these criteria were allowed to participate in the study. Sixteen of the 19 participants attended public schools; of these 16 schools, 3 were small (less than 700 students), 9 were medium-sized (between 700 and 1400), and 4 were large. The remaining 3 students attended small, private high schools.

All 19 students were quite successful in high school. The mean high school GPA for the 19 students was 3.84 (on a 4-point scale). Participants were also reasonably successful in mathematics. The mean ACT math score was 29; the mean GPA for students' 12th grade math course was 3.43. All 19 students took 4 years of mathematics in high school. Eighteen of the 19 students used "traditional" mathematics curricula during all 4 years of high school. One student (BS)[2] took 1.5 years of math using a reform curricula (Core Plus) during her first two years of high school and then switched to the "traditional" math track.

Sixteen students took AB Calculus in their senior year of high school; 2 students took Pre-Calculus, and the remaining student took AP Statistics (but did not take the AP exam). Of the 16 students who took AB Calculus, 6 did not take the AP exam. Of the 10 that did take the AP Exam, 1 earned a '5', 1 earned a '4', 3 earned a '3', 3 earned a '2', and 2 earned a '1'.

In the first semester at the University of Michigan, all 19 students took a mathematics course because doing so was a requirement for the major that each was considering pursuing. 5 participants were enrolled in PreCalculus (all 5 female); 10 were enrolled in Calculus I (6 males; 4 females); 4 were enrolled in Calculus II (3 males, 1 female). One student (AC, a female in Calculus I) dropped the course during the first semester, about half-way through the semester.

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[2] *We will be referring to each of the participants in our study with a two-letter code.*

In the second semester, 13 of the 19 students enrolled in a second semester of mathematics. (The 6 who did not take another math course cited several reasons for their choice, including a lack of interest in math and the fact that their anticipated major only required a single semester of math.) All 5 students from PreCalculus continued on to Calculus I; 7 of the 10 Calculus I students took Calculus II; and one of the 4 students in Calculus II continued to Math 215 (Differential Equations).

Participation. Participation in this study involved two kinds of activities. First, members of the research staff observed participants' Pre-Calculus and Calculus classes and homework groups. We observed all participants' classes at least once per semester, except in two cases where particular instructors preferred not to be observed. Homework groups were also observed once per semester. Observations were documented with detailed, written field notes.

Second, we conducted a broad range of data collection activities on the experiences of participating students. This second category included the following. First, students were expected to keep a math journal, in which they wrote about their experiences in their math class. Students were asked to write in their journal twice per week. Second, students were expected to complete two survey instruments. We assessed students' learning strategies with the Motivated Strategies for Learning Questionnaire [MSLQ] (Pintrich, Smith, Garcia, & McKeachie, 1993). In addition, we assessed students belief about mathematics using the "Conceptions of Mathematics Inventory" [CMI] (Grouws, 1994; Grouws, Howald, & Colangelo, 1996). Each student completed each survey twice, once in October and once in March. Third, students reported all mathematics grades, including scores for homework, quizzes, tests, midterms, and final exams. We also collected students' grades from high school and from standardized college entrance exams. Fourth, students were interviewed two or three times during the semesters in which they were enrolled in mathematics. (In the second semester, those students who were not enrolled in mathematics were only interviewed once.) All interviews were semi-structured and were tape-recorded. During interviews, students were asked to talk about their experiences in high school and university math.

Results

We begin the presentation of our results by describing what U-M Calculus classes looked like, based on our observations and field notes. We look closely at students' achievement in college mathematics and categorize 4 different patterns of achievement change. We then discuss students' perceptions of differences between high school and college mathematics. We conclude by connecting students' perceptions of difference with the patterns of achievement and attempt to draw conclusions about students' mathematical transitions.

Typical U-M class

Members of the research team observed U-M math classes and took detailed field notes. We observed 3 Pre-Calculus classes (all in the first semester), 14 Calc I classes (9 in the first semester and 5 in the second semester), and 6 Calc II classes (3 in the first semester and 3 in the second semester). Structural features of each observed class were first tabulated, including the composition (numbers and gender) of each class, the arrangement of tables and chairs, the start/end time of the class, and the attendance rate.

Next, the field notes were used to create categories of classroom actions. These categories included whether or not the class started or ended late, when and for how long students worked in groups (and what they worked on while in groups), when and for how long an instructor lectured (and on old or new material), and when students were assessed. These categories were then used to go back over the field notes and code the sequence of actions in each class. Finally, the collection of coded action sequences were analyzed, in an attempt to see if a portrait of a "typical" U-M class could be generated.

Although there was some variation in what classes looked like in each course, our analysis indicated that there was a great deal of uniformity among the classes we observed. With the caveat that we only observed a small fraction of the Pre-Calculus and Calculus classes at U-M in 1999-2000, we were able to create a composite picture of what our participants typically experienced in their math classes.

Classes typically lasted 80 minutes (90 minutes minus a 10-minute passing period). In the classes we observed, males instructors outnumbered female instructors by a factor of 2, while the student gender balance was about half female and half male. The 80 minutes of class time typically consisted of the following activities (see Table 1 for average number of minutes on each activity for each class): a series of announcements and collecting/passing out of work ; reviewing of the homework/quiz/exam problems by the instructor at the blackboard; group work to practice material covered in a previous class, for an upcoming exam, or new material; and a lecture on new material.

<u>Table 1</u>
<u>Average time (minutes) spent on various class activities in a typical class for each course</u>

	105 (n=2)	115 (n=10)	116 (n=4)

Start time	(on time)	(on time)	1 (late)
Announcements/Hand out papers	8	7	1
Review of homework (Instructor at board)	32	23	6
Group work to practice material	13	27	33
Lecture on new material	15	13	34
End time	9 (early)	6 (early)	(on time)

The averages in Table 1 do not include "atypical" classes, such as when a quiz was given (4 classes: three 115 classes and one 116 class) or when the instructor devoted the full class to reviewing for an upcoming assessment (3 classes: one each for 105, 115, and 116). Average time for each activity type for these atypical classes is given in Table 2.

Table 2
Average time (minutes) spent on various class activities for atypical classes for each course

	Quiz Class (n=4)	Review Class (n=3)
Start time	1 (early)	
Announcements/Hand out papers	3	5
Quiz	30	
Review of homework/quiz problems (Instructor at board)	18	80
Lecture on new material	12	
Group work to practice new material	10	
End early	6	

Student achievement

Almost all of the students' overall GPAs dropped in their first semester at the University of Michigan. Participants' mean first semester GPA was 3.17, a drop of 0.67 points from the high school mean GPA of 3.84. Individually, seventeen of the 19 students' GPAs dropped. The largest drops were 1.75 (DF) and 1.5 (LB). The only two students whose GPAs rose were SL (a rise of 0.2) and KK (a rise of 0.11). This drop in students' grades between high school and college is not unexpected and has been noted elsewhere in numerous studies on the transition to college.

A similar drop in students' grades shows up in students' achievement in mathematics classes. Students' mean GPA for their senior year of high school math was 3.43. The mean GPA for participants' first semester of U-M math was 2.98, or a drop of 0.60 points from the high school mean. Eleven of the 19 students had lower grades in their first semester U-M math class as compared to high school 12th grade math (largest drop was 2.0 points by DD and DF). Four students' U-M grades were the same as their 12th grade math grades, and 3 students' U-M grades were higher (highest rise was DB, 1.0 points). (One student, AC, dropped her first semester math course.)

Thirteen of the 19 participants took math again the second semester. Of the 11 students who reported grades to us[3] (mean first term math GPA for these 11 students was 3.19; mean high school math GPA was 3.56), the mean second term math GPA was 2.74, an 0.45 point drop from the first term (and a 0.82 point drop from high school). Seven of the 11 students' math grades dropped (largest drop: 1.3 points by JP and JV). Two students' grades stayed the same as in the first term, and 2 students' grades rose (highest rise: 0.7 points for TM and DD).

All of the above data on students' achievement for high school, first, and second semesters is shown in Figure 1.

(Insert Figure 1 about here)

Categories of student achievement.

Looking beyond the aggregate achievement results, individual participants show several patterns of achievement change in their mathematics classes. Examining individual results allows us to begin to understand students' experience in U-M Calculus, as each student's perception of mathematical discontinuity will likely be influenced by whether or not he/she felt successful, pleased, frustrated, and/or disappointed in his/her grade. Each achievement category will be used (see Locating mathematical discontinuities, below) as a way to interpret students' perceptions of and reactions to differences between high school and college math.

(Insert Figures 2 and 3 about here)

Figures 2 and 3 show a representation of changes in students' grades from high school to college, both for their mathematics classes and more generally. Based on Figures 2 and 3, we have divided the 11 participants who took mathematics for both semesters of their freshman year into the following achievement categories. Within each category, students accounted for or explain the changes in their achievement in different ways (see Figure 4).

(Insert Figure 4 about here)

Steady all year ("No Strugglers"): Three students (BS, SL, EB) were able to earn high grades with relatively little drop (or even a small gain) in their math grades during the year; we refer to these students as "no strugglers". Despite the fact that most students

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[3] *Two students did not report their grades in the second semester.*

in our sample experienced drops in their grades from high school to college, these four students (BS and SL in 105/115, EB in 116/215) held steady.

BS began the first semester well but suffered a relatively small drop in her grade in the second semester. BS was a very serious student and she took this small drop (from a 3.7 to a 3.3) very seriously. She indicated two main explanations for this decline. First, she felt that the course material was more challenging in her second term course and contained less review of the material that she covered in high school. Second, BS felt that her extracurricular life in college (she was a varsity athlete) took up an increasing amount of her free time.

SL was able to achieve high grades in both of her first year math courses (her second term grade of 3.7 was the highest among all participants). SL indicated that she was very challenged by the material throughout the year, and, in order to be successful, she pursued two strategies. First, she decided to hire a private tutor, employed him for the entire year, and met with him regularly (about once per week). Second, SL said that she put forth special effort in math. When she realized that the course would be particularly demanding, she made it a priority and was determined to get a good grade.

The third "no struggler", EB, attributed his success to an excellent high school math background and a strong work ethic. He was never very challenged by his coursework in math, and was the only student in our sample who took math beyond Calc II in his freshman year.

Good start but then major trouble ("Delayed Strugglers"). Also going against the trend of experiencing a grade drop from high school to U-M, four other students (MB in 105/115; DB, JP, and JV in 115/116) got off to a relatively good start in the first semester: MB and JP's first semester grades were the same as their high school grades (4.0 for both) and DB and JV's grades went up (1.0 and 0.8 points, respectively). However, after this initial success in the first term, all four of these students ran into major trouble, suffering significant declines in their math grades in the second term. We refer to these 4 students as "delayed strugglers": each of these students' second term math grade was over a letter grade lower than their first term grade. In interviews and in journals, these 4 "delayed strugglers" cited a variety of reasons to account for this decline. All 4 students mentioned that the course material in their second term course was harder and contained less review of high school topics than their first term course. Two students (JP, DB) commented specially on their second term instructor, complaining that he/she was less open to questions and less understandable. Two students (JP, MB) felt that they became very busy in the second term, and that their other courses and extracurricular activities made it more difficult to devote blocks of time to working on math. One student (JP) felt that his poor performance on the final exam was a significant reason why his grade was so much lower in the second semester.

Rough start but recovered ("Recoverers"): Two students (TM in 105/115, DD in 115/116) had a very rough start with their first term math course (experiencing grade drops of 1.7 and 2.0 points respectively), but each managed to bounce back and show significant improvement in their second term grade. We refer to these two students as "recoverers". DD's grade dropped from a 4.0 in high school to a 2.0 in his first semester. However, he was able to improve his grade to a 2.7 in the second term. DD cited three reasons from this recovery: he felt his GSI in the second semester was more open to questions and more understandable; he attended extra help sessions more regularly in the second semester; and he asked his peers for help more frequently. TM, whose 4.0 in high school fell to a 2.3 in the first semester and then rose to a 3.0 in the second semester, felt that the main reason for her improvement was that she became generally more acclimatized to college life.

Rough start but never recovered ("Strugglers"). Two students (LB in 105/115, MC in 115/116) had rough first and second semesters, as reflected in their math grades. LB, who struggled somewhat in high school math (her GPA in her 12th grade year was 2.5), had even more trouble with U-M math, earning a 1.7 in both first and second terms. MC's 4.0 in high school dropped to a 2.7 in the first semester and continued to drop to a 2.3 in the second semester. Both students felt that the first term was a rough one for them, and each made a special effort to improve their math grades in the second term. But interestingly, despite the fact that neither was able to show improvement, both indicated a belief that the second term had gone better than the first. Both claimed to be exerting a greater individual effort and both commented on a better (more approachable and understandable) second term GSI. In addition, LB commented that her group homework sessions were much more productive and useful to her in the second term than they were in the first term.

Perceived dimensions of differences

Our research questions focus on what students noticed as different between their high school and their college math courses, and how students adjusted to these differences. We first present several types of perceived dimensions of differences that students reported. We then match the achievement categories above with the perceived differences and try to more completely understand students' experiences.

Our analysis indicates that students did notice several dimensions of difference between their high school and college mathematics classes. These differences fall into four broad categories: (a) changes in responsibility for completing work and attending class; (b) the use of group work, both in and out of class; (c) changes in the types of problems on homework, quizzes, tests, and exams; and (d) a new focus on providing written explanations to accompany problem-solving steps. We use students' own words whenever possible to illustrate each of these types of perceived differences. Figure 5 indicates which students mentioned which differences.

(Insert Figure 5 about here)

<ins>Changes in responsibility</ins>. Several students (5 of the 11) commented that the locus of responsibility for completing homework and attending class, as well as other academic responsibilities, changed from high school to college. EB commented that in high school he could count on his teachers to make him learn; in college, he had to "teach himself". MB said that, "I've learned to teach myself, you know, like to do more reading and to try to understand it and do problems on my own" (10/1/99 Interview). MC commented that keeping up with the work of the class was not that important to her success in high school math; however, keeping up was imperative in college math. JV noted that he needed to read the textbook in college in order to understand the material -- something he never had to do in high school. In high school, JV said, he "learned through the teacher" (12/16/99 Interview). However, at U-M, he learned the 115 material primarily from the textbook.

While this perceived difference may be related to the curricular shift that students experienced, it is also a commonly noted feature of the high school to college transition generally.

Group work. A second difference between high school and college math was the presence of group work. The Harvard consortium curricula (as well as the U-M Mathematics Department guidelines on the teaching of 105, 115, and 116) mandate that group work be used regularly. Almost all of our sample of participants had very limited experience working with groups in high school math and thus noticed this feature of U-M math as unusual. Typically, students were assigned to their first group and then had some degree of choice in the composition of the remaining groups. (Group membership typically changed once or twice per semester.)

For many, group work was not an enjoyable experience. Participants indicated that students in a group often did not share goals for how groups should function or how the task (completing a homework set) should be accomplished. Students reported that they were given little advice from their mathematics instructors on how group homework should be done, other than "in groups". Students reported that some instructors recommended students assign each other roles within the group (such as scribe, recorder, explainer, etc.). Our participants indicated that this scheme was followed by very few groups (even fewer after the first couple of assignments) and also rarely enforced. Many students also reported that their group subverted the group homework experience entirely by either: dividing up the group homework problems and then completing them individually; or letting one person do all of the group homework problems by him/herself.

MC complained throughout the semester that she "would rather do it [the group homework] on my own" (10/4/99 Interview). SL commented that, "My group isn't very cooperative and moves on before everyone understands the solutions (11/1/99 Journal). SL also said that, "I don't really like the way my group works together because no one else looks over our group problems before we meet. This misuses our group time because we should be asking the group our problems with the work, not working the problems out together ... it's frustrating" (11/1/99 Journal). MB commented that what she learned from doing group work is that you cannot depend on group members, as many do not care or do not want to learn. According to MB, if you wanted a high grade, you had to work harder individually to achieve that grade.

However, even those students who complained about group work recognized its (theoretical) benefits. MC did find it beneficial to have difficult problems explained to her by a peer. EB, who enjoyed group work and felt it was beneficial to his learning, said of one of his groups, "We have been helping each other understand the material and I think that is why we work so well together" (11/12/99 Journal). LB noted that, "I think the idea of group work is good ... it is supposed to help you understand things better" (11/8/99 Journal).

<u>Different types of problems</u>. A third difference between high school and college was a change in the types of problems typically done in math class. Most students commented that U-M math courses did many more 'story problems' than were done in high school math. Often these story problems were quite challenging, as students had difficulty figuring out how to go about solving them. Referring to a particular test in his 115 class, JV said that, "the challenging part of this test is sorting out the story problems

and free response questions to be able to determine what the real question is" (11/10/99 Journal). DD had similar problems on a test: "I really studied and knew all I thought I had to know for the test. There [were] a few parts where I didn't know the algebra, but [one] problem threw me in particular... by the time I figured out the wording, it was too late. I felt confident in my skills over the section, however, my grade didn't reflect that" (11/11/99 Journal).

Similarly, MB commented that she did a greater variety of problems in her U-M math courses, and this diversity of problems made her think differently about the math involved: "[in college math], once you do a problem with it, basically you should be able to do all problems with it, concerning that formula, or that theory" (11/20/99 Interview). MB also said that in high school, the problems tended to be the type that "you used a basic formula and just applied it. With word problems they're not always all the same, so you don't know how, you don't know where you're supposed to put everything" (3/26/00 Interview). BS, who took 1.5 years of math using a reform curriculum (Core Plus) at the beginning of her high school years before choosing to switch to a more traditional track, reported (a bit unhappily) that the reappearance of story problems in U-M math felt like "an extension" of Core Plus.

<u>Providing written explanations</u>. As a required part of their math courses, students had to provide written explanations to accompany their symbolic solutions to homework and test problems. Many students commented on this requirement. For almost all, providing written explanations was initially a challenge. MC commented that in high school, most of the problems that were done were "one-liners", but in college, "you have to explain, and why you did this, and how, and for what reason..." (10/4/99 Interview).

DB felt that the emphasis on explanations was excessive and, while he saw the point in it, he still felt it was a bit "drastic" and did not "particularly feel that math class should be graded on your ability to verbalize it [your answers]" (10/8/99 Interview). LB thought that doing explanations were "a waste of time" and felt very frustrated by having to "explain something that I feel like I've known forever" (10/7/99 Interview).

However, as students became more proficient in providing written explanations, many began to feel that being forced to write about their solving methods made them think about the mathematics in a different way. JP contrasted the way he was doing math at U-M with the way he was used to doing math in high school by saying, "it's more of understanding the concept than working them out, say, algebraically" (10/26/99 Interview). DB felt that someday he would be in a situation where it would be useful to be able to explain a math problem to someone else, and he was glad to have learned this ability in 115. LB commented that providing explanations meant that a student "must also know what something is and explain how to apply it... The level of understanding in college is much deeper" (12/15/99 Interview).

Locating mathematical transitions

By cross-referencing the perceived dimensions of differences with the categories of student achievement, it may be possible to learn more about whether or not U-M students' experienced a mathematical transition during the 1999-2000 year. Figure 6 shows the four perceived dimensions of differences for each of the 4 categories of student achievement.

(Insert Figure 6 about here)

The first three categories (change in locus of responsibility, group work, and change in the types of problems) seem to be unrelated to student achievement (see Figure 6). Whether or not a student found any of these categories to salient does not predict which achievement category the student would fall into. There is no clear pattern; for example, MB noticed all three dimensions and was a "delayed struggler"; DB did not notice any of them and was also a "delayed struggler".

In contrast, observe how the different achievement categories viewed the role of explanations. The only students who did not mention that providing explanations was a new or salient aspect of their U-M experience were the students who ultimately did well in U-M math. None of the "No struggle" students, who earned high grades in both semesters, and none of the "Recoverers", those students who had a rough first semester but ultimately improved, mentioned having to provide explanations as a difference between high school and U-M math.

We are hesitant to draw conclusions from this finding, but we find it intriguing. Those students who did mention explanations as a perceived difference indicated that providing them required a "deeper" understanding of the mathematics, as LB said. Yet those students who had the most success in their U-M math courses (and ostensibly were the ones who understood the content the most) did not find explanation-providing to be a particularly salient or significant difference between high school and college math. One of our research questions asked how students' learning of mathematics was affected by the experience of a mathematical transitions. It may be the case that having to provide explanations in a reform-oriented curricula is one perceived differences that does affect the learning of students. We hope to pursue this issue in more depth in our next year of data collection.

Conclusions

Our analysis of the first year of data has begun to tell a story about U-M students' experiences in reform Calculus. Many U-M students struggled (in terms of grades) as they moved from high school to college; some were able to recover from this initial trouble while others were not. Students cited a number of reasons to account for their struggles or recoveries. In making the transition from a traditional high school math program to a reform-oriented one, most students noticed differences between these curricula. Four differences that figured prominently were the presence of group work, a change in the locus of responsibility, the different types of problems, and the need to provide explanations. There is some initial evidence that some of these transitions may have played a role on students' success or lack of success in terms of grades.

With these promising beginnings from our Year 1 analysis comes a recognition of the challenges that we face in our continued research. We list several of these challenges. First, we realized this year that U-M draws upon a very successful and specialized population of high school students (typically, those with high GPAs, 4 years of mathematics, and lots of AP courses); we need to give thought to how the extremely successful math backgrounds of the students in our sample may affect both our results and our ability to integrate our findings with those of the other 3 research cites. Second, we have gained an appreciation for how difficult it is to track the full experience of a relatively large number of students. We will need to give thought to how we improve our data collection efforts so that we can find out as much as possible about U-M students' experiences in math classes and in college generally. Finally, we have found our job complicated by the limits on students' course taking in mathematics. Only about half of our original Year 1 sample of 19 students took mathematics for a full year[4]. We will need to increase our sample size to adjust to this kind of attrition.

Our efforts in Year 1 have also raised a number of new questions that we hope to explore in more depth. First, we are interested in exploring the long-term effects of the Harvard Calculus program at U-M. For students who take mathematics courses beyond Calc II or 116, the curriculum goes back to a more traditional one. Do students experience this curricular shift as another mathematical transitions? In what ways is this transition related to the one which occurred as students moved into the Harvard courses? Also, we are interested in learning more from the students who stopped taking math. How do their reflections on the experiences in U-M math change over time?

Second, we have focused our analysis on students' achievement in U-M math and have proposed that students necessarily filter their experiences through their perceptions of success or failure in terms of grades. Yet we were struck by students such as LB and MC, who expressed some pride and satisfaction at the improvements they felt they had made in the second semester, despite the fact that their grades continued to drop. We would like to know more about students such as these, whose experience of the transition may be tied more to subjective factors rather than exclusively to grades.

Third, as we continue to learn more about the transitions that students experience, we are interested in finding out how different groups may play roles in helping students

[4] *And were reliable participants in this study. Although 13 of the 19 took math for 2 semesters, 2 students had to be dropped from the study*

navigate any difficulties that arise. How does a University, a department, a class, or a homework group contribute to students' negotiation of mathematical transitions? What resources offered by these different types of groups do students make use of, and how affective are these resources?

We have just begun to assemble our participants for our second year of this endeavor, and we look forward to building upon these initial findings in the year ahead.

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for non-participation.

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