Students' experiences moving between "traditional" and "reform" curricula: What are the implications for K–16 mathematics education?

John P. (Jack) Smith III
Michigan State University

Jon R. Star
Michigan State University

Amanda Jansen Hoffmann
Michigan State University

Paper presented at the NCTM Research Pre-Session, Las Vegas, NV, April 2002

Session Discussants:
Jo Boaler (Stanford University) & Sarah Thuele-Lubienski (Iowa State University)

Acknowledgements. In addition to Smith, Star, and Hoffmann, the rest of the Mathematical Transitions Project staff played vital roles in the work reported here. These other team members, in alphabetical order, are Dawn Berk (University of New Hampshire and Michigan State University), Carol Burdell (Michigan State University), Beth Herbel-Eisenmann (University of Wyoming), Amanda Jansen Hoffmann (Michigan State University), Violeta Lazarovici (Michigan State University), and Gary Lewis (Michigan State University). We also acknowledge the fine work of our supporting graduate and undergraduate student analysts at Michigan State University and the University of Michigan: Cindy Bardel, Alissa Belkin, Glenda Breaux, Rachel Catt, Matthew Dilliard, Adrienne Frogner-Howell, Andrea Kaye, Abby Magid, Kelly Maltese, Brian My, William Nham, Kristy Rider, Sharon Risch, Courtney Roberts, Samantha Spatt, Brian Walby, Heidi Walson, Tracy VanOs, and Shannon Wellington. The project is supported by a grant from the National Science Foundation to the first author (REC-9903264).

Correspondence should be addressed to Jack Smith, jsmith@msu.edu, 442 Erickson Hall, Michigan State University, East Lansing, MI, 48824. A copy of this paper is available on-line at www.mathtransitions.msu.edu.

Students' experiences moving between "traditional" and "reform" curricula: What are the implications for K–16 mathematics education?

Part I: Introducing the Question

For many teachers, educators, researchers, and parents, the 1989 National Council of Teachers of Mathematics <u>Curriculum and Evaluation Standards</u> (and the documents that it inspired) expressed a powerful and compelling vision for mathematics teaching and learning (cites). Right or wrong, mathematics itself appeared as a more open subject-matter; learning a more creative, exploratory, and autonomous process; and teaching a matter of discovering and shaping students' initial ideas. The emergence and the broad acceptance of the Standards has in turn initiated an active period of curriculum development, implementation (spotty, but national in scope), and research and evaluation (cites). But since the Standards vision differs markedly on many dimensions from prior, also broadly accepted view of mathematics, learning, and teaching, shrill and painful debate has also emerged, e.g. "the Math Wars" in California (cites). A national consensus on the goals and methods of mathematics education is not yet at hand.

In this period (roughly 1990 to present), little attention has been given to students' reactions to and experiences of the curricula developed to embody and enact the Standards vision. This is not to say that "students" are not cited in debates or that student performance and attitude data has not been collected to evaluate the impact of Standards-based curricula. It has (cites). But citations of students' experiences in debate are typically journalistic in nature; they are included to support the position that the partisan has already staked out and are often extreme. By contrast, evaluation data tend to examine student outcomes without much attention to students' experience—particularly, how they see, make sense, and feel about the changes that have taken place in their mathematics classrooms.

The project that generated the evidence and arguments in this paper was initiated to address this deficit. The Mathematical Transitions Project, based at Michigan State University (MSU), is a three-year, NSF-funded project that examines students' mathematical transitions at four sites (2 high schools and 2 universities). At each site, students move between programs with "traditional" expectations for mathematical work and those with expectations more consonant with the NCTM Standards (in short, "reform" curricula). At two of the sites (one high school and one university), students move from a "traditional" curriculum to a "reform" curriculum; the reverse (a move from "reform" to "traditional") is true at the other two sites.

In this paper, we pose a fundamental question that expresses an assumption underlying our entire work. Answers to this question govern how the work of projects like this one might support the search for a national consensus on the goals and methods of mathematics education.

> *Should research on the experiences of students who move between "traditional" and "reform" curricula influence future directions for K-16 mathematics education?*
>
> *If so, how?*

If the answer to this question is "No," the National Science Foundation should reconsider its decision to fund research of this sort. Further, different "Yes" answers are possible, particularly with respect to how research might support dialogue, development, and consensus building. Our nearly 3 years of hard work indicate that we think that the answer is "Yes." Moreover, we have given some thought to how our results, and others like them, might be useful. But it is not the views of a few that count. What does count is a broader discussion and digestion of these ideas by the many. We hope that this paper and the professional gathering it was written for will be part of such a process.

But before we begin, a major caveat. We have gathered extensive data and partially digested the experiences of some 80 students in our 4 research sites. But the analysis of this extensive corpus is not yet complete. For this paper, we present and use as a basis for discussion the experiences of 8 students—2 from each research site. Besides our intentional choice on a male and female student in each pair, the selection of the 8 "examples" was essentially random (see below for details). Thus the data that we present to support our points in this paper is quite modest. Given that one of our basic findings is that students' reactions and experiences differ (in particular, there is no single reaction to reform), generalizing anything from these 8 cases would be illogical and dangerous. We can only ask that our readers and session participants hold this central limitation in mind as they read and participate.

Stances on the Question

Before we describe how we have operationalized "students' mathematical experience" and present the experiences of our 8 students in that framework, we "seed" the discussion of the fundamental question posed above by considering 5 possible answers to it. These "answers" draw on our experiences reading and listening to the local and national debates on value of the Standards-based reforms.

> *<u>The "Killer Anecdote" Position</u>[1]: "Yes, of course students' experiences are relevant, and oh by the way, here is the student example that illustrates/proves my position."*

This stance has already been alluded to above. One possible rationale for a "Yes" answer is to support arguments based on a single, usually extreme case. We have found that both proponents and critics of the reform movement have argued by using student example (cites). Though this approach can be very effective rhetorically (hence it's wide use in journalism), it is only logical if students' experiences do not vary in any important way. Hence, the implicit assumption of this stance is that <u>my</u> example is representative. That is, if <u>my</u> student had these experiences, then most (if not all) others must have too. Such "killer anecdotes" can be very effective in debate contexts when one's opponent is not ready with their own. But of course when they are, deadlock is imminent: No feature of one example can logically undermine the power of the other. We find it interesting that the generation of contrasting (often polar opposite) examples does not seem to lead combatants to a more productive adjusted position. If students' experiences do differ, maybe

[1] The term is not ours. We are grateful to Jim Fey for using it in a past conversation. He may or may not agree with how we are characterizing the nature of a "killer anecdote." Such ideas are our own.

we should expect this diversity and examine students' experiences accordingly. But then humans often never let data spoil a good theory!

> <u>*The "It's Too Complicated" Position*</u>*: "No, we really can't —though it sounds good in principle. There is just too much diversity in students' experience to pay attention to this factor. We need to decide directions on other grounds."*

This position is the conceptual dual to the Killer Anecdote stance. Here wide and intractable human diversity is asserted. In the extreme, every student's experience is different and therefore no groupings or patterns should be expected. On this view, research may be well-intentioned but is essentially futile. Moreover, if one accepts this extreme diversity, any patterns (even partial ones) reported in research become suspect. They would reflect poor research design (e.g., poor sampling) or researcher bias (e.g., ignoring data selectively) or both. The message to researchers here is "Don't bother." The lesson for the national dialogue is, "Students' experience cannot be a sensible source of information for shaping a national consensus on mathematics education; adult experts will have to do that." Note here that any empirical pattern in the experiences of a significant number of students undermines this stance, presuming that pattern and the research it emerges from passes methodological muster. The argument that empirical research on students' experiences is pointless can be undermined empirically.

> <u>*The "Just Rely on Teachers" Position*</u>*: "Maybe, but you don't have to go to the trouble of gathering information from students. Teachers are even better at gathering and interpreting information on students' mathematical experiences. After all, teachers spend more time than anyone trying to understand their students' lives and experiences."*

This third stance is both a short-cut away from the complexity of researching students' experiences directly and a valuing of teachers as interpreters of that experience. On its face, it is neutral with respect to students' experience of reform and traditional curricula and teaching. But a little thought shows that this neutrality exists only on the surface. After all, teachers often have substantial say in which curricula get taught in their schools, and they certainly have direct control on <u>how</u> those curricula get taught in their classrooms. So, advocates of this position must address the question of how teachers might be chosen to offer their sense of their students' experience. Different teachers, with different stances toward traditional and reform curricula, will certainly produce different accounts of students' experience. Moreover, this stance does not explicitly acknowledge that some important issues for students could escape teachers' attention entirely. With some warrant, it is hard to reject this "invisibility" as either impossible or implausible.

> <u>*The "Wisdom of Tradition" Position*</u>*: "No, students' experiences in the short-term really doesn't matter; they will eventually grow into an appreciation for mathematics (or not) as it has been traditionally seen, taught, and learned. We should respect the generations of human inquiry that has built up mathematics enough and teach students to respect (and learn) that tradition."*

This fourth stance also answers "No," but for a different reason than "It's too complicated." This position is (implicitly at least) suspicious of the Standards vision and reform movement. Mathematics, it asserts, is an important body of cultural knowledge, built up over many centuries from work in many countries and national cultures. The way that that mathematics is seen and practiced in <u>the</u> mathematical culture (that is, the community of practicing mathematicians) provides all the guidance needed to develop a national consensus around goals and methods in mathematics education. Whatever is going on with the vision of mathematics, teaching, and learning in this reform movement, it is—at best—an introduction to the way mathematics is <u>really</u> done. Examining students' experiences when they have not yet entered this culture will be misleading at best. Better yet to ask the best mathematicians we can find to define our national goals and methods. We note two basic problems with this stance. First, it assumes, against a fair amount of evidence, that mathematicians' views of their discipline, how it is learned, and how it should be taught, are unitary. How do we judge <u>the</u> tradition if it is not unitary? Second, if too many students never get to see "what mathematics is really about" because they choose to terminate their mathematics education early, this stance could be, by definition, true for the minority and false for the majority.

> *<u>The "It's Challenging" Position</u>: "Yes, students' experiences are relevant, but we should not expect easy or simple lessons from these experiences; 'students' do not think and experience in the same or even similar ways."*

This last position is a more qualified and problematic "Yes" than the Killer Anecdote stance. It acknowledges the wide diversity cited in the "It's Too Complicated" position but sees that diversity as less extreme and, potentially at least, subject to careful inquiry. In particular, it leaves open the possibility that patterns in students' experience are possible, even if those patterns do not hold for all students. But if research on students' experience is sensible and perhaps even desired, simple-minded designs will not work. Effort must be extended to do justice to the diversity of student experience, both across students and across the range and dimensions of that experience. Experience is more than a matter of performance and attitude. Students as sense-makers are forming judgments of what mathematics is (and should be) while they study it in school. This is the stance that we have taken in our work, and we hope to defend our conceptualization of "students' mathematical experience" and research design as a reasonable implementation of it.

Part II: Introducing the Project

In this section we provide a conceptual and methodological overview of the Mathematical Transition Project (MTP)—one sustained effort to study students' experiences of Standards-based (and traditional) mathematics curriculum and pedagogy. This brief overview aims to (1) address the challenge of the complexity of students' experience that is implicated in the "It's Challenging" stance and (2) introduce the 8 case studies that follow. (For more detailed accounts of conceptual and methodological issues; sites, curricula, and participants; and students' experiences (beyond the 8 presented below), see our previous publications, which are available at www.mathtransitions.msu.edu.)

We consider the framework we have developed to study students' mathematical experience an important result of this work, quite independent of the experiences of particular students framed within it—either these 8 students or the 80 students we have "followed" in our work. Others could, for example, usefully adapt and/or apply this frame as is in other school settings, other curricular contexts, and with other age children. That said, we understand that—however likely or unlikely its use by other researchers may be—readers are interested in what this frame allows us to see and claim about our students.

Basic Design

We have been interested in tracking the impact of Standards-based curricula on students' experience of mathematics. Given "spotty" implementation, students can move into and out of different reform curricula. So our research design was built around two different curricular changes: Students can (1) move from traditional into Standards-based curricula, and (2) move from Standards-based into traditional curricula (for brevity, the <u>T → R shift</u> and the <u>R → T shift</u>, where "R" stands for "reform" and T for "traditional").

Next, we chose to study these shifts at two points in students' mathematical development: Entrance to high school and entrance to university. We recognize that significant changes in students' mathematical experience can and do also take place in the elementary and middle school/junior high years. Our focus on high school and college was driven primarily by: (1) limited resources of time, money, and personnel, (2) a lack of knowledge of local elementary-junior high school site pairs with the "right" curricular shifts, and (3) recognition that much of the "heat" in the Math Wars has come from shifts in high school and college mathematics curricula. We also considered but elected not to examine the also interesting T→T and R→R shifts for reasons of limited resources. Though such potential pairings lacked a significant shift in curricular vision, they could be very interesting as contrasts for the R → T and T → R shifts examined here.

Third, we chose research sites (2 high schools and 2 universities) and particular reform curricula based on what was regionally available. Our sites are briefly described below. The three main reform curricula were: (1) the Connected Mathematics Project (CMP) middle school materials (at the R → T high school site), (2) the Core-Plus (CPMP) Project high school materials (at the T → R high school site and at the R → T university site), and (3) the Harvard Consortium Calculus (HCC) materials (at the T → R university site). Last, we chose target numbers of participants and the overall duration of our study. We followed approximately 20 students at each of the 4 sites across 2.5 years of schooling in mathematics. In general, we had little direct knowledge of their prior mathematical experience and had to uncover that, post-hoc, in our interviews.[2] Our basic design is summarized in Table 1, below.

At the Michigan State site, in addition to working with graduates of the CPMP high school curriculum from around the state, we also recruited graduates of one other high school (pseudonym "Hartville High") where teachers designed and implemented their own Standards-based mathematics curriculum. Their curriculum development took functions as a central

[2] Fortunately, we did have extensively prior knowledge of students' experiences and classroom practices at one site, Prescott High (e.g., Herbel-Eisenmann, 2000).

organizing concept and developed it across the Algebra I-Algebra II-Pre-Calculus-Calculus sequence.

Table 1. The 2x2 basic design of our project

	Type of Curricular Shift	
	Reform to Traditional	Traditional to Reform
Junior high to High school	Prescott High (PHS) CMP to Traditional	Logan High (LHS) Traditional to CPMP
High school to College	Michigan State (MSU) CPMP & Hartville High to Traditional	U. of Michigan (U-M) Traditional to Harvard Calculus

Conceptualizing "Mathematical Experience"

We have taken this broad and vague term and segmented it into 4 dimensions or slices of experience. However imperfect this analytic segmenting may be, we see some such scheme as necessary to study students' mathematical experience empirically.

First, we consider students' <u>achievement</u> in school mathematics to be an important factor and have operationalized "achievement" in the usual terms, grades in mathematics courses. Tracking change over time is relatively straightforward on this dimension: Does the students' achievement change significantly after a curricular shift?

Second, we hypothesized that what students saw in the curricular shift was an important part of their experience. If mathematics educators argued so much about the relative merits of traditional and reform curricula, it would be surprising indeed if students saw them as similar in all salient respects. So we sought information on <u>what student saw as different</u> in their "new" curriculum.

Third, following MacLeod (1992), we combined students' beliefs and their attitudes toward mathematics in a single dimension, <u>disposition towards the subject</u>. Tracking change here is also relatively straightforward (at least conceptually): Do students' beliefs and attitudes about mathematics change after a curricular shift? In particular, does the curricular shift appear to influence their interest in and enjoyment of mathematics?

Last, we were interested in exploring the relation between the curricular shift and <u>how students went about learning mathematics</u>, which we refer to as "learning approach." Under learning approach, we group all actions and strategies students choose to undertake learn mathematics. In turn, we asked: Does the student's learning approach change after a curricular shift?

Combining these four dimensions, our conceptualization of mathematical experience can be summarized as follows:

$$\text{Students' Mathematical Experience} = \sum(\text{Achievement} + \text{Differences} + \text{Disposition} + \text{Learning-Approach})$$

and thus

$$\text{Changes in Students' Mathematical Experience} = \sum(\Delta\,\text{Achievement} + \Delta\text{Differences} + \Delta\text{Disposition} + \Delta\text{Learning-Approach})$$

In this conceptualization we assume that important correlations may well exist between these dimensions (e.g., change in achievement co-occurs with a change in disposition towards the subject) and expect that our empirical analysis would uncover and shed light on them. Our overall aim has been to determine which students experienced transitions within their mathematical experience and why. To make this determination, we have defined a <u>mathematical transition</u> as <u>significant change in two or more of the four dimensions</u>. We hope to make the case below that this general definition makes sense in terms of the specific experiences of students—at least in 8 cases. (See Smith & Berk, 2001, for a more detailed presentation of this conceptualization of "mathematical experience.")

Data Sources

Generally speaking, we conducted the following data collection activities at all 4 sites. We (1) conducted classroom observations (to assess the "taught" curriculum), (2) collected information on students' grades, (3) held individual interviews (2 to 3 per semester on average), (4) administered surveys of beliefs about mathematics and learning, (5) collected weekly student journal entries, and (6) asked students to solve problems from their texts as well as problems we designed. Information on differences noticed came from interviews and journals; information on disposition was drawn from interviews and belief surveys; and information on learning approach came from interviews.

Methods of Data Analysis

Our richest source of data was the individual interviews. In general, our analytical method had two steps. First, for each dimension that we assessed through interviews (e.g., differences noticed, disposition, and learning approach), we developed and tested a coding scheme that served to reliably identify utterances that fell within a particular dimension. Two project members independently coded each interview; separate passes over the same transcript were needed, one for each dimension. No team member coded interviews they conducted. The two coders then met to compare their codes. In small numbers of cases, coders could not agree on a particular statement, but these instances were very rare (after resolution discussions). We refer to this first step as "objective" coding.

Following this process of objective coding, one team member was charged with determining (more qualitatively) whether the collection of coded statements in a particular dimension (in conjunction with other materials[3]) showed that the student's mathematical experience had been "significantly" affected by the curricular shift or not. This qualitative analysis was presented to and discussed by the entire team, until the group reached consensus on a yes/no decision for each student. Each dimension was considered independently in this phase of the analysis. For example, we attempted to determine whether a student had experienced significant change in her disposition without attending to prior or pending decisions concerning change in achievement, learning approach, or disposition.

Together, these two steps in our analysis enabled us to produce a "scorecard" for each student. This score card indicated whether a student did or did not experience significant change in each of the 4 dimensions. (For all dimensions, we allowed for the possibility that a student's utterances did not provide sufficient evidence to determine significant change.) More detail about how we determined significant change is presented below.

For <u>Achievement</u>, we examined change in mathematics course grades, semester by semester, both alone and in light of overall academic performance. The simplest way to do this would have been to examine the difference in mathematics grades over each successive time period. However, simply comparing two consecutive math grades seemed overly simplistic and not sufficiently sensitive to changes related to the curricular shift in mathematics. For example, some students may have experienced a drop in all of their course grades, due to academic challenges of a more general nature, rather than due to mathematical difficulties alone. To take such possibilities into account, we decided to use the following method to compare achievement in mathematics from semester to semester. We found the change in math grades between consecutive semesters and compared it to the change in GPA, by subtracting. We call the result of this computation the "relative change" (RC) -- see formula below. A positive relative change in a student's math grade indicates an improvement in the student's math grade relative to the grades in other courses taken during that semester. A negative relative change indicates a decline in math grade, relative to those in other courses taken

$$\boxed{\text{RC} = |\text{Math grade (before)} - \text{Math grade (after)}| - |\text{GPA (before)} - \text{GPA (after)}|}$$

If the resulting difference was greater than |0.5| (on a 4 point scale), we concluded that the relative change was significant. 0.5 points seemed a reasonable criterion on which to base this decision. (See Appendix 1 for the grades and RC values of the 8 students analyzed in this paper.)

Within the <u>Differences</u> dimension, our objective coding yielded for each participant a list of which differences were mentioned in which interview. Almost all students pointed to at least one aspect of the new curriculum that was different. But some students mentioned many differences (and some did so in multiple interviews), while others only rarely mentioned differences. In order to determine whether a student noticed "significant" differences in the curricular shift, we came up with the following rubric: A student noticed significant differences if s/he mentioned 3 or more differences consistently across at least half of his/her interviews.

[3] For example, toward the end of the project we asked some students to draw a graph of their interest in and enjoyment of mathematics over time, starting the year before the project began; we refer to this task as the "Disposition Graphing" task. Because this task was a very late addition to our methodology, not all students were able to complete such a graph.

Two additional pieces of information were used to support this decision on differences. First, in at least one interview, most students were specifically asked to characterize their new mathematics program as either "very different," "somewhat different," or "not at all different" from their old mathematics curriculum. To earn a rating of "noticed significant differences," a student who was asked this question must have responded either "very different" or "somewhat different." Second, some students mentioned differences that thematically linked or clustered around a particular issue but in a way that our coding scheme somewhat artificially separated into discrete units. If a student mentioned a cluster of thematically linked differences consisting of at least 3 differences, and if this cluster was mentioned in at least half of all interviews (even though each differences may not have been mentioned this frequently), we considered the student to have noticed significant differences. (Examples of such conceptual clusters will be given below.)

We determined significant change in Disposition by looking carefully at five different types of statements that fell within our category of disposition: Attitude (like or dislike of the mathematics; engagement or motivation in mathematics), self-efficacy, preferences (e.g., for a different kind of mathematical experiences—including their former program), emotion (anger, irritation, etc.), and career plans. (Our objective coding sorted disposition statements into these five categories.) Significant change in disposition required the determination of explicit change in more than one coding category. We especially attended to attitude and self-efficacy as they seemed the most direct and potent indicators of disposition. Most of the disposition statements came from the interviews, but we also asked many students (including all the high school participants) to construct Disposition Graphs. On these graphs, students located points indicating the level of their interest in and enjoyment of mathematics before and after the curricular shift. For students who completed Disposition Graphs, we compared the graph to the thrust of the interview analysis.[4]

Likewise, with Learning Approach, we looked for explicit changes in more than one category in a somewhat longer list of features in our operationalization of this dimension: Physical participation (in class), intellectual participation (in class), textbook use, graphing calculator use, engagement with homework, preparation for classroom assessments, use of others, resource management (time and space to learn).

Once significant change had been assessed in all four dimensions for a student (and the "scorecard" generated), we classified those students who experienced significant change in 2 or more of our 4 dimensions as having experienced a "mathematical transition."

Our Example Students: The "NCTM 8"

Recall that we have chosen to present our analysis of 8 out of our overall sample of approximately 80 students. There are two essential points to make about how we chose these 8 students. Since 1/10 is a small fraction of our overall sample, we run the risk of having our

---

[4] We were not able, due to limitations in our project resources, to collect Disposition Graphs from many participants, particularly students at U-M.

intentions and claims misinterpreted, so we feel it important to be clear about this selection process.

First and foremost, our choice of students was essentially random, subject to a few constraints. We wanted one boy and one girl from each site. Furthermore, since students' interviews were being continually transcribed, we selected students for whom at least the first 4 interviews transcribed. From the short list of students who data fit these two constraints, we chose randomly. We did not choose based on pre-existing ideas of what these students' stories were.

In fact, our second criterion was that we had not yet developed any top-level story for them. We have spent much of our second and third project years developing, debating, and revising our sense what we mean (operationally) by each of our four dimensions of students' mathematical experience. We viewed the writing of this paper as an opportunity to take our analytical method for a test-drive. We began this process with no assumptions about what our conceptualization would yield for particular students. We selected our sample of 8 randomly so that we could get a true sense of whether our process "worked."

That said, what is important to know in advance about these 8 individuals and the schools they attended? We offer a brief introduction to each student, organized by site. We have selected pseudonyms consistent with the students' gender. (See Appendix 2 for a list of which mathematics courses these 8 students took during each term that they participated in our research.)

At <u>Prescott High School</u> (PHS), a R→ T site, students moved from three solid years of Connected Mathematics (CMP) into a relatively traditional "layer-cake" curriculum using Glencoe, Prentice-Hall, and UCMSP textbooks. At PHS, we selected Suzanna and James for this analysis. Both were successful with CMP in the middle school and chose to take Geometry as 9th graders, thereby skipping Algebra I.[5] Both went on to take and complete Advanced Algebra as 10th graders. Suzanna volunteered for the project as an 8th grader (based on a description of the project at the end of that school year); James volunteered in the middle of 9th grade, when we recruited a few more students to round out our sample. Though there were many similarities between these students, we found they had quite different experiences in the high school. In the Prescott district (as well as in the T→ R high school site, Logan), one and only one middle school (junior high) "fed" into one and only one high school. Thus, at both sites, all the students knew all of their high school peers.

At <u>Logan High School</u> (LHS) where students moved from a traditional junior high school program (7th and 8th grades) into Core-Plus (T → R), we chose Rachel and Devin. About half of our site sample at LHS were introduced to Core-Plus as 8th graders when they chose Course I over the alternative of Algebra I at the junior high.[6] To balance our site sample, we recruited

[5] About half of each cohort of rising 9th graders at PHS take Algebra I, while the other half takes Geometry. Placement is currently governed by student (and family) choice; 8th grade mathematics teachers give advice but do not make placement choices.

[6] This was certainly a glitch in our research design (we missed the transition into CPMP for half our sample at LHS, since this shift occurred before we began interviewing students at this site), but our design requirements (T→R) made finding even one site within driving distance of MSU difficult.

about half our students from this "advanced" group; the other half were introduced to CPMP (that is, took Course I) as 9th graders. Both Rachel and Devin were in the "advanced" group and took Course II in Spring of their 9th grade year. In contrast to the standard year -long courses at the junior high, Logan High utilized “block” scheduling, and each semester of mathematics presented about 2/3 of a year of content. Within the block schedule, the 4 year CPMP curriculum was segmented into 5 semester courses (Course I-V), and students could “double up” if they wanted (take mathematics in both Fall and Spring semesters). Rachel and Devin both took a lot of mathematics during the study. Rachel took mathematics in 5 consecutive semesters (including the Spring semester of 9th grade) by doubling up as a 10th and 11th grader (Course II-V and AB Calculus). Devin doubled up during his 10th grade year, taking 4 mathematics courses (Course II-V).

At Michigan State University (MSU) where we studied graduates of CPMP and Hartville High’s reform curricula in traditional college courses (R→ T), we selected Melissa and Dean, both intending engineering students.[7] Melissa was a Hartville graduate who placed into Calculus II at MSU as a result of her AP Exam score. In her final two years of high school mathematics, Melissa took Pre-Calculus and AP Calculus at Hartville, courses with many features of the Standards-based reforms. A standard textbook existed in each course but was used sparingly and for specific and limited purposes; extended projects were a central element to one; motion of objects observed in class was the basis for mathematical exploration in the other; oral and written explanations of solutions was expected in both. Dean was a graduate of Core-Plus who placed into Pre-Calculus at MSU as a result of Placement Test score.[8] Both took three consecutive semesters of mathematics beginning in the Fall of their freshman year (Melissa: Calculus II-IV; Dean: Pre-Calculus, Calculus I-II). Dean dropped Calculus II before completing the course. The MSU calculus sequence (including pre-calculus) is taught primarily in larger lectures (80-100 students) and smaller recitation sections (25-30 students). The exception is Calculus I, which is taught entirely in small sections (25-30 students) mainly by tenure-stream faculty.[9]

Finally, at the University of Michigan (U-M) where students from a variety of traditional high school programs studied introductory mathematics (Pre-Calculus, Calculus I & II) using the Harvard Consortium materials (T → R), we selected Becky and Frank. Becky came to U-M interested in international business and placed into Pre-Calculus in the Fall of her freshman year based on her Placement Test score.[10] She took and completed both Pre-Calculus and Calculus I. Frank, an intending engineering student, enrolled in Calculus I in the Fall of his freshman year, completed it, and enrolled in Calculus II the following semester. All sections of all three introductory courses at U-M are taught in small sections (25-30 students), primarily by graduate student instructors (GSIs). More experienced and/or talented instructors are typically assigned to Calculus I and II; instructors in Pre-Calculus are often just learning to teach. From the beginning of the implementation of Harvard curriculum, the mathematics faculty added a group homework

---

[7] Our sample included no mathematics majors, at either MSU or U-M. Our intending engineering students were those whose programs required them to take mathematics (Calculus I-IV).

[8] The MSU Mathematics Placement Test is composed to multiple choice items involving traditional algebraic manipulation skills.

[9] The commitment of faculty teaching resources to Calculus I at MSU is primarily an attempt to boost student retention in the calculus sequence.

[10] Like MSU, the U-M Mathematics Placement test consists primarily of multiple choice and short answer questions assessing algebraic manipulation skills taught in high school.

component to the standard (written) features of the Harvard materials. As one might expect, group homework is not a familiar activity for students from traditional high school experiences.

Part III: Eight Students' Experience of a Curricular Shift

In this section we report and interpret our characterization of these 8 students' experiences across 4 dimensions of a curricular shift in mathematics. Table 2 below presents a summary of our results. From a summary judgment within each dimension (yes = significant change; no = no significant change) we list an overall judgment of whether the student experienced a mathematical transition (or not) and a one-word characterization of the nature of that transition. Recall that significant change in two or more categories was necessary and sufficient for concluding a student experienced a mathematical transition.

Table 2. Summary of the "NCTM 8" Students' Mathematical Experiences and Transitions

Site (Shift)	**Student**	**Achievement Change?**	**Significant Differences?**	**Change in Disposition?**	**Learning Approach Change?**	**Transition?**
PHS (R→T)	James	No	Yes [cluster + freq]	Yes ↓	No	Yes Affective
PHS (R→T)	Suzanna	No	Yes [cluster + freq]	No	Yes	Yes Adaptive
MSU (R→T)	Melissa	No	Yes [cluster]	No	Yes	Yes Adaptive
MSU (R→T)	Dean	Yes↓ (1st sem.)	Yes [cluster + freq]	Yes ↓	Yes	Yes Both
LHS (T→R)	Rachel	No	Yes [cluster + freq]	No	No	No
LHS (T→R)	Devin	No	No	No	No	No
U-M (T→R)	Becky	Yes ↓ (2nd sem.)	Yes [cluster]	No	Yes	Yes Adaptive
U-M (T→R)	Frank	No	Yes [cluster]	No	Yes	Yes Adaptive

<u>Note</u>: in the Differences column, "cluster" stands for "thematic clustering" and "freq" stands for "frequency" of differences

Table 2 lists two quite different types of mathematical transitions, <u>Affective</u> and <u>Adaptive</u>. To qualify for an Affective transition, a student's disposition toward mathematics must change and the differences he/she reports must be related to that dispositional change. To qualify for an Adaptive transition, a student's learning approach must undergo significant change in a manner that addresses the differences he/she has reported. These types of transitions will be discussed further, below.

Broadly speaking, the "action" for these 8 students is captured by the differences they saw between their "old" and "new" program and the changes they made in their learning approach. All but one reported significant differences; five of 8 significantly changed how they went about learning the subject. Only 2 experienced significant changes in achievement (Dean and Becky), and 2 (James and Dean) experienced significant changes in their disposition towards mathematics. In all 4 cases, the changes were downward: Lower mathematics achievement relative to their academic performance and weaker enjoyment and interest in the subject. Six students experienced mathematical transitions in our terms, 4 adaptive, 1 affective, and 1 adaptive and affective. The greater number of adaptive transitions reflects the "action" in the differences and learning approach columns relative to the other two dimensions. We also note that all four R→T students experienced transitions, where only 2 of the T→R did (though small numbers make it impossible to call this a pattern, much less an interpretable one).

Before we turn to a closer individual examination of the students' experiences, we note one major departure of our conceptualization and results from other projects that have examined the impact of curricular change on students' mathematical experience. In most studies we have found (cites), impact has been analyzed in terms of achievement only or, in some cases, achievement and attitude. For these 8 students the impact of curricular change was <u>not</u> primarily located within those dimensions. It remains to be seen if this pattern will hold up in the larger sample. But for now, it is a demonstration of how misleading (and potentially dangerous) it can be to equate impact with effect on grades and attitude toward mathematics. There is much more to students' experience than those two dimensions, however important they are.

Prescott High School (R → T)

At <u>Prescott High School</u> (reform/CMP → traditional/various texts), the experiences of James and Suzanna form an interesting contrast. While their grades do not change much and they both observed differences between their middle school and high school mathematics experiences, they differed in their reactions to these observed differences with respect to impact on disposition and learning approach. In terms of achievement, neither student's grades changed much, in mathematics or more generally, from the 8th through the 10th grade. Suzanna was in the top 1% of her class, earning a 4.00 in all of her courses for all four semesters in her first two years of high school. James' grades generally slipped, but only somewhat in the 9th grade across all of his courses, including mathematics, from an A– in 8th grade to B+ and B's in high school. Overall, they were both good students in school who did not experience major changes in their grades.

Both students reported significant differences between CMP and their high school math classes (Geometry; Prentice-Hall; Advanced Algebra, UCMSP). These differences were numerous (Suzanna observed 8 differences; James noted 12.) and clustered around common features—how typical problems and problem solving had changed in their "new" program. For Suzanna, high school problems were more "complicated," involving more steps to reach a final answer. The differences she noted might be among those expected from any observant student moving from one mathematics class to another regardless of the curricular shift, such as the decrease in the applied nature of high school problems.

> Yeah, they were...they don't...they show you examples [in the CMP texts], but they don't have you actually solve the examples. I was looking at my sister's math book the other day, and they have a big, huge problem that they have to solve, and then they have little problems that go along with it that might not be exactly with

> it. It might be different numbers and stuff. The Geometry book, they just give you examples and then have questions to sort of relate to the...They don't have one big set problem, one big set of word problem that you have to figure out, because most books, I think in eighth grade were really in context. They always had a problem to go along with it, and it was always set up that way. They wouldn't just give you numbers, and tell you to solve for *x*. They do that in our book now a lot. There would be a problem that said solve for *x* when *y* = -2, and *x* = -3, and stuff like that. (Suzanna, 10/19/00 Interview)

She also discussed whether the textbook had worked out examples or answers in the back of the book (the high school text had both, the middle school text did not), but did not indicate the presence or absence of these examples as being a problem or an asset for her.

For James, the change had more to do with his expected contribution as a learner. Whereas he had to determine formulas for himself in middle school, high school teaching gave him the formulas and asked him to "regurgitate" them.

> But, the homework is more repetition than ... understanding and stuff. And, the tests are easier, I think. Not really easier, but there is not as much stuff to write. 'Cause like before when we had a test, it would be like a paragraph on why Sally can't walk to school. And I said, I have a question, but right now it is just like, find the area of the sphere. It is easier stuff, but we never, we go over it and then like I don't know. I think my problem is I just make mistakes and ... 'Cause like when it is three-fourths pi or whatever times radius squared, I always like mess up. (James, 5/17/00 Interview)

This example illustrates that that he noticed how the curricular materials, such as assignments and tests, expected less of him intellectually.

In terms of disposition, James' more negative attitude toward math was related to this altered role: He sensed his value in the classroom had decreased, and he expressed frustration about this. Affective statements were tagged onto his description of differences, such as "I thought that was really cool" (James, 2/22/00 Interview) in reaction to his description of getting to work with other students in structured ways in 8^{th} grade. He said, "I had more fun last year. I'll go back to 8^{th} grade." (James, 5/17/00 Interview) Also, he related this lack of involvement to a decrease in the expectations that he construct his own knowledge, saying "We don't have to think as much." (James, 5/17/00 Interview). In Suzanna's case, the differences she noticed did not seem to affect her disposition towards mathematics as a school subject. When she spoke about differences she observed between middle school and high school mathematics, her reactions were mostly neutral in terms of affect. Although Suzanna observed differences, they did not seem to impact her attitude.

James gave no evidence that he changed how he went about learning mathematics. Neither in 8^{th} grade nor thereafter did he do much homework at home, nor did he deeply engage in small group work. In contrast, Suzanna's learning approach did change as she moved from reform into traditional mathematics. She reported using her graphing calculator less in high school and for more lower-level purposes (e.g., checking her calculations) as well as taking more homework home. But she also reported that she worked more with peers in class than she did in 8th grade, where she worked primarily by herself. However, there is an important caveat here: She worked primarily with one student throughout the entire 9th grade year, because they were in the same class and were good friends outside of class. She then turned to a more solitary approach in 10th grade.

Both James and Suzanna experienced a mathematical transition as they moved into traditional mathematics. We see James' experience as an Affective transition because the "new" expectations for his mathematical activity in class were closely associated with his decreased engagement intellectually and emotionally. (Note that this connection is one that we draw; James did not give us evidence that he was conscious of it.) Suzanna experienced an Adaptive transition because she simply adjusted to the differences she saw, committing more of her own time to learning the content and working with her friend in class. We see the impact of the curricular shift more strongly in James' case; he was not happy at all with the role of being the "recipient" rather than "generator" of mathematical knowledge.

Michigan State (R ←F)

Melissa and Dean's experience at Michigan State replicate some aspects of James and Suzanna's experiences but with different results and in different combinations. Melissa, who experienced two years of "reform" mathematics at her high school, followed the broad outlines of Suzanna's experience. She was an excellent student (before and after the curricular shift), saw that collegiate mathematics was different in important ways, and made adjustments in what she did to learn accordingly. Similar to Suzanna, Melissa generally characterized her "new" program as "very different" from her old one. Like James, she noticed that the role for students in learning mathematics was different in college:

> I mean, in high school they had lots of time to make you develop your own, like they made you develop your own theory in your head. Like your own way of understanding it. Like they developed it. But like here they just give it to you. And you are expected to understand it the way they know it. And I mean I am fine with it. I may not remember it as long. (Melissa, 1/25/01 Interview)

Melissa adjusted to the increased pace of presentation and the difficulty of asking questions in the lecture context by seeking out a context—the Math Help Center—where she could ask questions and work through the content at her own pace. Though she valued a classroom where students were expected to develop their own understanding, she accepted the move into traditional mathematics and "presented methods," and her disposition toward the subject was not strongly affected. She still liked mathematics, liked learning it, and continued on her Mechanical Engineering career path.

Dean, who graduated from 4 years of CPMP, had a quite different experience. Unlike all the other students in the "NCTM 8," his mathematics achievement was significantly affected in the first semester, dropping from an A in 12th grade to a B (or 3.0) in Pre-Calculus in the first semester of his freshman year. He maintained that B in Calculus I in the Spring semester (many freshmen at MSU do not) but dropped Calculus II without completing it in the following Fall semester. Since he entered with an interest in engineering, dropping Calculus II involved a change in overall academic direction. By the middle of his sophomore year, Dean expressed his intent to change his major.

Dean saw many differences between Core-Plus and his MSU mathematics classes, and these differences seem directly related to the change in his disposition towards mathematics. He contrasted the more relaxed pace and classroom climate, the in-class group work, and the realistic application problems in Core to the faster pace, solitary work, and focus on symbols in his college classes. The increased focus on symbolism seemed related to his access to the content:

> Yeah, more symbols, and it is more like, everything is written out in mathematical terms, where as my high school math would be less formal. More, kind of personal. Well, not personal, but less formal. (Dean, 3/2/01 Interview)

Dean's disposition toward collegiate mathematics changed significantly. Where mathematics had been his favorite subject in high school, he found college mathematics boring and began to feel that he was less able mathematically. However, he drew a distinction between mathematics as a subject and the collegiate classes he was taking: His declining interest was only a result of how mathematics was taught at MSU; it did not apply to mathematics per se. The differences he found with high school also led Dean to change his learning approach as well. Because he had found mathematics relatively easy in high school, he did not adjust to the different demands of his collegiate courses. His use of his graphing calculator reduced to merely checking answers, because calculators were not allowed on tests; where he did no homework in high school, he undertook it in a spotty way in college; he found his college textbooks harder to read for the content; and without wide access to group work in class, he sought but was not successful in establishing peer study groups away from class.

We characterize Melissa's experience, like Suzanna's, as a <u>Adaptive</u> transition, indeed a successful one. She faced the changes and addressed them without heavy losses or struggle. Dean's experience of mathematics changed on all dimensions.[11] He tried to adapt to the changes that he saw, but was not successful, either in terms of his achievement or his disposition towards the subject. That he still liked the subject generally was a positive outcome, but one that helped little to learn mathematics at MSU.

Logan High School (T ←R)

At <u>Logan High School</u>, Rachel and Devin moved into the Core-Plus mathematics program as 8th graders from a more traditional program in 7th grade and before. (Recall that about half of our Logan sample moved into Core-Plus in 8th grade—those in the "advanced" math track—and half in the 9th grade when they came to the high school.) Both were recruited in the Spring of their 9th

---

[11] We note that the label, "Both Adaptive and Affective" transition, may or may not appropriately capture all that changed in Dean's experience.

grade year,[12] Rachel in Course III and Devin in Course II. Neither student's mathematics grade changed much either when they moved into Core-Plus as 8th graders or during their 9th grade year. Rachel's grade dropped 0.70 points from Course III to Course IV when she complained that the content was getting harder. Devin's grade rose slightly from 7th to 8th grade (B+ to A–), remained there for Course II and III, and dropped slightly in Course IV (back to B+).

Likewise, neither students' disposition toward the subject changed significantly in high school. They had different orientations toward school (and mathematics): Rachel was a "classic" good student who was disciplined in all things. Devin liked mathematics well enough but was much more engaged in sports than school. It appeared that neither student's interest or enjoyment changed across the curricular shift. With respect to learning approach, we could detect no change in how they went about learning their mathematics. They appeared to be doing more or less what they had always done. Devin's work was primarily limited to what happened in class; he devoted only some attention to homework at home. Rachel was more conscientious, working hard in class and at home.

Both students noticed differences between Core-Plus and their prior mathematics work, especially the Pre-Algebra course they took as 7th graders. But only in Rachel's case did these differences meet our criteria for significance. Both noticed that the Core-Plus problems were typically set in a "real life" context, were different, one to the next, and therefore could not be solved by a single procedure that was specified in the text. Rachel articulated some of these differences in the following terms:

> Well, in Math 1 [CPMP Course I], you do more solving and it's more written stuff and more experimenting things at first and doing different tables and stuff, and, like, when I was in pre-algebra, it was just, like, you learned how to figure out the problem and then you just applied it to different numbers, you know." (Rachel, 5/9/01 Interview)

Neither Rachel or Devin met our criteria for an overall mathematical transition. Devin did not earn a "yes" score on any of 4 dimensions. The only difference between Rachel and Devin was primarily that Rachel mentioned more differences that clustered around a particular theme. Devin was a man of few words and did not easily elaborate or repeat his views. In that respect, he serves as a reminder in our work that one element of diversity is how much students are prepared to and/or disposed to talk about their mathematical experience.

University of Michigan (T → R)

Our final two students, Becky and Frank, were successful graduates of traditional high school mathematics programs before they entered the University of Michigan's introductory program as freshmen. Both took mathematics in the Fall and the Spring of their freshmen years. Becky took Pre-Calculus as a junior in high school and then AP-Statistics as a senior. Though she scored well enough on the U-M Math Placement test to enroll in Calculus I, Becky opted for the more

[12] Logan was the last research site to be located and as a result no data collection took place in the Fall of 1999 when work began at the other sites. With the exception of two students (Rachel being one) all our Logan began their high school mathematics work in the Spring semester of 2000.

cautious approach of taking Pre-Calculus first. Her intended career in international business required at least one semester of calculus, so she continued on to Calculus I in the Spring. She earned an A in Pre-Calculus in the Fall (as she had in AP-Statistics the year before) but found Calculus I more demanding, and her grade slipped to a B–. Becky saw connections between her work in economics and calculus, but her experience in both courses had her re-evaluating her career choice by the end of freshman year.

Frank took AP-Calculus as a senior in high school but did not score well enough to place into Calculus II (his AP score was a 2). So he took Calculus I in the Fall semester, and his mathematics grade dropped dramatically from an A in high school to a C at U-M. This drop did not earn him a "change in mathematics achievement" because his overall GPA dropped almost as much, from 4.08 to 2.33. So his achievement in mathematics <u>did</u> change (a steep drop) but not in a manner that was singularly mathematical. Frank struggled with his college coursework generally. As an intending engineering student, he pressed on to Calculus II in the Spring but chose to reduce his participation in the project to a single interview. In this one interview we learned something about his sense of Calculus II, but we do not know how well he performed in that course. In that sense, Frank represents an unfinished story.

Both Frank and Becky noticed differences between their U-M and high school mathematics programs; both generally characterized U-M as "somewhat different" from high school. Frank's differences clustered around the character of problems, the lack of repetition among U-M problems, and the focus on understanding what and explaining in one's own terms. In part he characterized this difference as follows:

> Like in high school is was just problem[s], just do them, find the answer. Now it's related to other things, you know. That was a big one, and in high school the teachers, you know, pound it into your head that you know how to do the problem, how to do it. Now they just focus on what you are going to get, what's your answer going to give you, and stuff like that. (Frank, 3/26/00 Interview)

In this passage, Frank struggled to express clearly what the "other things" were at U-M. But in an earlier interview, he contrasted the "cookbook" mathematics of high school with the "conceptual stuff" that U-M instructors focused on.

Becky, by contrast, focused more on instructional differences in the Fall semester. She disliked what she considered the lack of clear and explicit instruction (presentation of the content), the instructor's perceived inability to answer her questions effectively, and the absence of effective classroom management (too many students "off-task"). As she put it in a 11/20/99 interview, "In college because it was, I just found myself teaching myself more than the teacher standing up there and teaching me." This challenge however was temporary; she found her Calculus I instructor in the Spring semester much more effective.

We did not find that either student changed significantly in their disposition towards the subject (though our time with Frank was truncated). Both were (and remained) generally positive. But as Becky's quote just above suggests, the weaknesses that she perceived in her Fall instruction led her to undertake a different and wider scope of learning activities. These included relying more on the textbook's presentation of the content, going to the Math Lab to seek answers to her questions, using her graphing calculator less (she worried she was too "dependent" on it), and taking a very strong and lead role in completing her group homework. Frank also made significant changes but was less successful with them. Chiefly, he complained that he had established the pattern in high school of not doing assigned homework (in mathematics at least), which he now found necessary element for success. He reported he was trying to complete the individual homework on a regular basis but was not yet successful in the middle of the Spring semester.[13] Indeed, he was frustrated that high school had not "forced" him into such a pattern.

Summary points from these cases

In contrast to other studies of the impact of Standards-based (or "reform") mathematics curricula (cites), our analysis adds at least two dimensions of students' experience: how they see the differences in past and present curricula, and whether they change how they learn mathematics in response to such changes. We hope that the 8 "cases" sketched above support two general (if preliminary) findings: (1) the qualitative dimensions of students' experiences are important complements to aspects of their experience we can quantify, and (2) even within the same "scorecard" of results, important differences in experience exist. With respect to the first point, in half the cases (Suzanna, Rachel, Frank, and Melissa) students experienced their "new" mathematics program very differently without having these differences show up in measures of achievement and attitude. Unless we want to define impact in terms of these two dimensions only, we need to look more carefully at students' experience. With respect to the second point, we characterized three students (Suzanna, Frank, and Melissa) as all experiencing an Adaptive transitions with no impact on achievement, but how they adapted, the degree of effort they expended, and the success they achieved in their adjustments all varied. In other words, classes of transitions may be useful analytic descriptors (e.g., Affective, Adaptive, etc.) but some aspects of students' experience will require analysis at the level of the individual.

Part IV: What Lessons Can We Draw from these Students?

First, it is necessary to begin with a disclaimer: Anyone, ourselves included, who would draw conclusions about the "impact" of Standards-based curricula on students on the basis of 8 cases is uninformed about statistics and psychology or is behaving unprofessionally. Our project goal in preparing this paper was to carry a complete set of analytic methods on a small set of students so that we could be better prepared to complete the job on our full sample of 80 students. That is our next task. What we hope these cases illustrate, beyond the methods employed, is some of the diversity we can expect in the larger sample. At this time, we are only confident of one broad result in the larger sample: Most students (though not all) notice and report significant differences. Beyond that, we cannot yet say (nor can anyone else).

Beyond this general caution, we can state some results that we think the 8 cases do support and some issues that their experience raises:

[13] At U-M, the group homework is graded and counts significantly towards the students' grade, where the individual homework is assigned and at times discussed but is not collected nor graded. Thus the U-M students must decide, for reasons other than impact on their grade, whether or not to do it.

Achievement and/or attitude alone are insufficient measures of impact. Data on students' achievement (mathematics course grades) and attitudes (via questionnaires) is easy to collect, so research on "impact" of this variety is relatively easy to carry out. But achievement and attitude alone capture only a part of students' experience, and taking the part for the whole can be a dangerous leap in any domain. We think we have shown that large sample studies that target only achievement and attitude will tell only a limited story about "impact." On the other hand, more detailed studies like this one will support only limited generalizations. Eighty students is a lot, but still a small number given four research sites and the evident human diversity that we have only sampled in this paper.

Expect diversity in students' responses to new curricula. If any result is supported by this analysis, it is that "students can react very differently" to the same curricular shift even at the same site. It follows that broad general claims that "students like this method" or "react negatively to that approach" are simply not warranted. Based on these results, there is no homogeneous collection of "students." One can only respond to such claims with questions like, "what sort of student are you thinking about?" Within each of the 4 pairs of students we found significant differences in how they react to the same curricular shift. What explains such differences? It is hard not to pose this question and ponder it, but in fact, it is also probably impossible to answer it with any clarity. Diversity in response could be primarily a function of individual psychological factors ("kids are just different"), a function of prior mathematical experiences, or more likely an evolving interaction of the two. Our point is simply that by 9th grade wide diversity exists in what attracts students, what is merely "OK," and what makes them recoil. Clearly this diversity puts pressure on any effort to engage students' experience as a consideration in shaping our mathematics education goals and systems.

Students can be resourceful and adaptive, but getting through is not always "success." Generally speaking, the ability to adjust to changes in one's environment is a good thing—even when that environment is harsh and challenging. Similarly, we think that students show certain strengths when they emerge from one sort of mathematics education program, land in another quite different one, and manage to "get along," if not profit in their new program. So other things being equal, the ability to adapt to changing circumstances is desirable. Beyond this general consideration, when students move along in their mathematics education, they move through educational settings with increasing mathematical status. High school teachers are generally understood to know more mathematics than junior high/middle school teachers, and college professors much more than high school teachers. So each successive adaptation, one could argue, carries higher value. Successful adjustments to collegiate mathematics are "worth" more than successful adjustments to high school mathematics. But are all adjustments to mathematical environments positive? Not necessarily. If environments present to students a mix of mathematical virtues and shortcomings, then adaptation can be a mixed blessing. We need to look carefully at what our students gain as well as lose at the next level of their mathematics education.

Part V: Return to the Questions

We framed this paper and conference session around two questions:

Should research on the experiences of students who move between "traditional" and "reform" curricula influence future directions for K-16 mathematics education?

If so, how?

Currently in the United States, we have a highly decentralized system for choosing mathematics curricula, supporting articulation between levels (e.g., elementary and middle school) and adjusting teaching and assessment to these choices. Similarly, we lack strong mechanisms for even considering how "research" might bear on these choices. Acknowledging those deficits, we believe that our questions are still relevant to U.S. mathematics education and that our answers to them can affect how we address choices of what to do next—even locally and even without effective mechanisms to communicate such research to decision-makers.

As we indicated in Section II, we believe that students' experience is relevant, despite the substantial diversity of students who experience these shifts. Diversity among students and diversity in their experience of school mathematics is a lesson to be learned and an opportunity to be explored, not a rationale for excluding their experience entirely. In closing we argue for this position in two steps: (1) diversity is real and non-trivial, and (2) despite the variation, there are important lessons to be learned. Consistent with our position, we draw on the results from the NCTM 8, and on one final point, evidence from the larger sample of students we have been studying.

As we have argued above, we should all expect diversity in students' responses to mathematics curricula that designers and users see as "fundamentally different." We see that diversity directly in the experiences of our 8 students presented here. We also see it in our larger sample of students, though—at this point in our work—in a more diffuse and preliminary way. But then if students' reactions to different curricula are "challenging" in their diversity, then does that diversity not become an argument for setting aside students' experience as a major consideration in how we think about and evaluate directions for K–16 mathematics education? Why should we not simply rely on pre-college teachers' judgments of students' experience or the collective wisdom of the mathematical community for overall direction?

Because we cannot yet speak from a complete analysis of our full corpus of data, this is not an easy challenge to address in a substantive way. But we think diversity is no argument for failing to explore students' experience directly. We continue to hold to the premise that important and potentially numerous patterns may emerge, but they will take careful work to ferret out. Our optimism is supported here by the commonality we see in these 8 students' experience and in many others in the larger corpus. Many (though not all) notice how problems differ across "traditional" and "Standards-based" curricula. They see differences in how problems are presented (e.g., primarily numerical or primarily in words); what sort of guidance is provided to solve them (e.g., general concepts or related problems or a well-defined and identified method), what sort of response is expected (e.g., a numerical answer only or an answer and a written explanation), and how these related differences make them think about mathematics as a subject and their role as a participant in that intellectual activity. This cluster of related differences arises in everyday classroom activity as problems are continually assigned and solved and over time can become students' view of mathematics and their interest and place in it. From one perspective, we are pointing here to one potential pattern that could hold across the diversity we argue for more generally. (And just to be clear, some students "like" one side of these problem-related differences and some the other.) But from another perspective we may have located a more general conceptual theme that could shed light on major chunks of our data across aspects of their mathematical experience. That is our hope and the near-term direction for our analysis. We’ll be back.

References

Herbel-Eisenmann, B. A. (2000). How discourse structures norms: A tale of two middle school mathematics classrooms. Unpublished doctoral dissertation, Michigan State University, East Lansing.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 575–596). New York: Macmillan.

Smith, J., & Berk, D. (2001). The *Navigating Mathematical Transitions Project:* Background, conceptual frame, and methodology. Paper presented at the annual meeting of the American Educational Research Association, Seattle, WA.

(remaining references will be added at a later date)

Appendix 1
Changes in Achievement by Semester, NCTM 8

		Before Shift		Term #1		RC[b]	Term #2		RC	Term #3		RC	Term #4		RC
Student	**Site**	**Math**	**Overall**	**Math**	**Overall**	**B-->T1**	**Math**	**Overall**	**T1-->T2**	**Math**	**Overall**	**T2-->T3**	**Math**	**Overall**	**T3-->T4**
James	PHS (R→T)	3.70	3.75	3.30	3.43	-0.08	3.30	3.62	-0.18	3.00	3.20	0.12	3.30	3.26	0.24
Suzanna	PHS (R→T)	3.70	3.92	4.00	4.00	0.22	4.00	4.00	0.00	4.00	3.95	0.05	4.00	4.00	-0.05
Melissa	MSU (R→T)	4.00	4.00	4.00	3.90	0.10	4.00	3.91	-0.01	4.00	4.00	-0.09			
Dean	MSU (R→T)	4.00	3.50	3.00	3.30	-0.80	3.00	3.25	0.05						
Rachel	LHS (T→R)	4.00	3.89	3.70	3.90	-0.31	3.70	3.67	0.23	3.00	4.00	-1.03	3.00	3.90	0.10
Devin[a]	LHS (T→R)	3.70	3.62	3.70	3.47	0.15	3.70	3.67	-0.20	3.30	3.43	-0.17			
Becky	U-M (T→R)	4.00	3.91	4.00	3.83	0.08	2.70	3.09	-0.56						
Frank	U-M (T→R)	4.00	4.08	2.00	2.33	-0.25									

[a] Devin did not take mathematics in one term. As a result, his grades in terms #1 and #2 are not in consecutive semesters (see Appendix 2).
[b] Relative change, or RC, is calculated by the following formula:

$$RC = |\text{Math grade (before)} - \text{Math grade (after)}| - |\text{GPA (before)} - \text{GPA (after)}|$$

Appendix 2
Course-taking by Semester, NCTM 8

		1998-1999 Year	1999-2000 Year		2000-2001 Year		2001-2002 Year
			First Semester	Second Semester	First Semester	Second Semester	First Semester
James	PHS (R→T)	CMP 3	Geometry (i)	Geometry (ii)	Algebra II (i)	Algebra II (ii)	n.p.[b]
Suzanna	PHS (R→T)	CMP 3	Geometry (i)	Geometry (ii)	Algebra II (i)	Algebra II (ii)	FST (UCSMP)
Melissa	MSU (R→T)	AB Calculus	Calculus II	Calculus III	Calculus IV	No Math	No Math
Dean	MSU (R→T)	Core 4	Pre-Calculus	Calculus I	Calculus II[a]	No Math	No Math
Rachel	LHS (T→R)	Pre-Algebra	Core 1	Core 2	Core 3	Core 4	Core 5
Devin	LHS (T→R)	Pre-Algebra	Core 1	No Math	Core 2	Core 3	Core 4
Becky	U-M (T→R)	AP Statistics	Pre-Calculus	Calculus I	n.p.	n.p.	n.p.
Frank	U-M (T→R)	AB Calculus	Calculus I	Calculus II	n.p.	n.p.	n.p.

[a] Dean dropped Calculus II midway through this semester.
[b] n.p. indicates students who chose not to participate in the project during that semester.