"Reform" at the collegiate level: Examining students' experiences in Harvard Calculus

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In the decade since the 1989 publication of NCMT's Curriculum and Evaluation Standards, curricula consistent with that vision for pre-college mathematics education have been written and implemented in many U.S. elementary and secondary schools. A related but distinct movement at the university level has led to the development and use of Standards-based Calculus curricula (Douglas, 1986; Steen, 1988; Tucker, 1990). Evaluation and assessment studies are beginning to focus on how these curricula perform relative to traditional curricula in supporting student learning and positive attitudes (e. g., Hoover, Zawojewski, & Ridgway, 1997; Schoen, Hirsch, & Ziebarth, 1998). Educators have also begun to examine the processes of teacher learning and change in implementing reforms (e.g., Lloyd & Wilson, 1998; Putnam, Heaton, Prawat, & Remilliard, 1992). But the implementation of these reform curricula has been often "spotty," nationally and regionally. Many school districts and universities have chosen Standards-based curricular materials for one or more levels of their K–16 system while retaining older curricular materials that reflect less of the Standards vision at other levels. Often, such non-systemic implementations reflect substantial differences, within communities and between school buildings, on how mathematics is best taught and learned.

These "spotty" implementations can create conditions where students experience very different expectations for what it means to think, know, and do mathematics. For example, one curriculum may value and reward students' ability to explain their thinking, work productively with other students, and undertake large-scale inquiry relatively independently, while the previous (or subsequent) program of curriculum and teaching may not. As students move between schools (and sometimes even within schools), such transitions between Standards-based and more traditional curricula are increasingly common. Yet very little attention has been paid to studying the effects of these potential shifts for students (cf. Smith, Star, Herbel-Eisenmann, & Jansen, 2000; Star, Herbel-Eisenmann, & Smith, 2000; Walker, 1999). As students move into and out of mathematics classrooms and curricula which reflect different expectations and ways of knowing, what do they notice? How are their learning and attitudes toward mathematics affected? How do they adjust to changes when they recognize them?

These are the sort of questions we address in the research presented in this paper. We have just completed the first year (1999-2000) of a three-year, NSF-funded project which examines students' mathematical transitions at four sites (2 high schools and 2 universities). At each site, students move between programs with "traditional" expectations for mathematical work and those with expectations more consonant with the NCMT Standards (in short, "reform" curricula). At two of the sites (one high school and one university), students move from a "traditional" curriculum to a "reform" curriculum; the reverse (a move from "reform" to "traditional") is true at the other two sites. The specific "reform"-oriented programs included in our study are the Connected Mathematics Project [CMP] (Lappan, et al., 1995), Core Plus Mathematics Project [CPMP] (Hirsch, Coxford, Fey, & Schoen, 1996), and the Harvard Consortium Calculus program [Harvard Calculus] (Hughes-Hallett, Gleason, et al., 1994). This paper focuses exclusively on one of the university sites -- the University of Michigan [U-M], where many students move from traditional high school programs to courses taught using the Harvard Consortium materials. (For a more thorough introduction to the project, see Smith & Berk, 2001.)

Perspective. The concept of "mathematical transitions" has not been examined in academic research, so we want to be clear about our basic terms and meanings. We call marked differences between students' prior notions of what it means to think and act mathematically and how they are expected to think and act in their current classroom "mathematical discontinuities". These discontinuities “happen” in and around some mathematical content, but they refer to students’ experience of new (or different) expectations for their activity with that content—not to the content alone. Mathematical transitions are students’ conscious experience of and responses to those discontinuities: how they experience and understand the difference(s), how they respond (or not) to them, and how they understand the results of these responses. Mathematical discontinuities and transitions are naturally occurring phenomena, but they become more likely in periods of curricular reform where “implementation” is uneven. (See Smith & Berk, 2001, for a more thorough discussion of mathematical discontinuities and transitions.)

Research questions. Our work centers around the following research questions. First, what do students notice as different in their current mathematics experience, as compared to what was experienced one year ago? Answering this initial question requires that we characterize and compare the intended (text materials) and enacted (teaching practices) curriculum in students' current mathematics classes with the corresponding curricula in their high school programs. Second, what mathematical transitions do students experience? Third, for those students who do experience transitions, what strategies and resources do they try out in an attempt to adjust to the discontinuities? What strategies and resources do they ignore? What are the consequences of trying out those strategies and resources? (See Smith & Berk, 2001, for a more thorough discussion of our research questions.)

Mathematical discontinuities at the University of Michigan. The University of Michigan is an ideal location to search for students' experiences of different mathematical expectations. Most U-M first-year students have taken 4 years of mathematics in high school and have used "traditional" mathematics curricula in all courses, up to and often including an AP Calculus course. Upon entering U-M, all students who choose to take math (other than those in honors courses) take courses which use a reform curriculum -- the Harvard Consortium materials. At U-M, the Harvard Consortium curricula are used in three, semester-long[1] courses: PreCalculus [Math 105] (Connally et al., 1998) and Calculus I [Math 115] and II [Math 116] (Hughes-Hallett et al., 1994). The Harvard Consortium materials claim to differ from more traditional curricula along several dimensions, including an emphasis on real-world and contextual problems, a greater focus on multiple representations of topics (geometric, numerical, analytical, and verbal -- “The Rule of Four”), the development of formal definitions and procedures from work on practical problems (“The Way of Archimedes”), and an increased depth of understanding rather than breadth of coverage.

All three U-M courses are taught in many small sections of approximately 25-30 students. Although each section of each course is taught independently by a single instructor, all sections within a course share common homework assignments, common unit tests, and a common final exam. Group work is required in and out of class. In class, students typically sit at tables in groups of 4 and are often encouraged to work on problems with those sitting around them. Out of class, students are assigned a group and are given a homework assignment every week to be done in groups. This group homework assignment is typically composed of problems which are more difficult than those the student might see in class or on tests, ostensibly to encourage more of a group effort.

[1] In this paper, we use "semester" and "term" interchangeably. We call the term which runs from September to December the "first" semester or term and the one that runs from January to April the "second" semester or term.

After the Calc II (Math 116), students who continue taking mathematics move on to an introductory course on Multivariable Calculus or Calc III (Math 215). Math 215 and all subsequent courses following Math 116 do not use the Harvard or other reform-oriented materials.

We begin by describing our data collection method and the students who chose to participate in our study. We then characterize these students' experience in the U-M Calculus program, in terms of the four criteria which we use to determine whether or not students experienced a mathematical transition: achievement, approach to learning, disposition, and perception of difference (for a more thorough description of our criteria for determining a mathematical transition, see Smith & Berk, 2001).

Method

Participants. Nineteen first-year students at the University of Michigan volunteered for this study (10 females; 9 males). Students were recruited by posting flyers in the building where their mathematics classes met; students were compensated $250 per semester for their participation in this study. In order to eligible to participate, students had to be enrolled full-time as first-year students at U-M, over the age of 18, and have attended high school in the state of Michigan. All students who volunteered and met with these criteria were allowed to participate in the study. Sixteen of the 19 participants attended public schools; of these 16 schools, 3 were small (less than 700 students), 9 were medium-sized (between 700 and 1400), and 4 were large. The remaining 3 students attended small, private high schools.

All 19 students were quite successful in high school. The mean high school grade point average [GPA] for the 19 students was 3.84[2]. Participants were also reasonably successful in mathematics. The mean ACT math score was 29; the mean GPA for students' 12th grade math course was 3.43. All 19 students took 4 years of mathematics in high school. Eighteen of the 19 students used "traditional" mathematics curricula during all 4 years of high school. One student (SB)[3] took 1.5 years of math using a reform curricula (CPMP) during her first two years of high school and then switched to the "traditional" math track.

Sixteen students took AB Calculus in their senior year of high school; 2 students took Pre-Calculus, and the remaining student took AP Statistics (but did not take the AP exam). Of the 16 students who took AB Calculus, 6 did not take the AP exam. Of the 10 that did take the AP Exam, 1 earned a "5", 1 earned a "4", 3 earned a "3", 3 earned a "2", and 2 earned a "1".

In the first semester at the University of Michigan, all 19 students took a mathematics course because doing so was a requirement for the major that each was considering pursuing[4]. 5 participants were enrolled in PreCalculus (all 5 female); 10 were enrolled in Calculus I (6 males; 4 females); 4 were enrolled in Calculus II (3 males, 1 female). One student (CA, a female in Calculus I) dropped the course during the first semester, about half-way through the semester.

In the second semester, 13 of the 19 students enrolled in a second semester of mathematics. (The 6 who did not take another math course cited several reasons for their choice, including a lack of interest in math and the fact that their anticipated major only required a single semester of math.) All 5 students from PreCalculus continued on to Calculus I; 7 of the 10

[2] All grades will be given using the traditional 4-point scale, where an A is a 4, a B is a 3, a C is a 2, and a D is a 1.

[3] We will be referring to each of the participants in our study with a two-letter code which does not correspond to each students' actual initials.

[4] Those students who major in Business, Pre-Med, or Accounting are required to take math up to and including Calc I (Math 115). Engineering majors have to take 4 semesters of Calculus.

Calculus I students took Calculus II; and one of the 4 students in Calculus II continued to Math 215 (Calc III).

Participation. Participation in this study involved two kinds of activities. First, members of the research staff observed participants' Pre-Calculus and Calculus classes and homework groups. We observed all participants' classes at least once per semester, except in two cases where particular instructors preferred not to be observed. Homework groups were also observed once per semester. Observations were documented with detailed, written field notes.

Second, we conducted a broad range of data collection activities on the experiences of participating students. This second category included the following. First, students were expected to keep a math journal, in which they wrote about their experiences in their math class. Students were asked to write in their journal twice per week. Second, students were expected to complete two survey instruments. We assessed students' learning strategies with the Motivated Strategies for Learning Questionnaire [MLSQ] (Pintrich, Smith, Garcia, & McKeachie, 1993). In addition, we assessed students belief about mathematics using the "Conceptions of Mathematics Inventory" [CMI] (Grouws, 1994; Grouws, Howald, & Colangelo, 1996). Each student completed each survey twice, once in October and once in March[5]. Third, students reported all mathematics grades, including scores for homework, quizzes, tests, midterms, and final exams. We also collected students' grades from high school and from standardized college entrance exams. Fourth, students were interviewed two or three times during the semesters in which they were enrolled in mathematics. (In the second semester, those students who were not enrolled in mathematics were only interviewed once.) All interviews were semi-structured and were tape-recorded. During interviews, students were asked to talk about their experiences in high school and university math.

Results

We begin the presentation of our results by describing what U-M Calculus classes looked like, based on our observations and field notes. This section is intended to provide an introduction to the U-M Calculus program and also to provide some support for one trend noted in the project overview paper (Smith & Berk, 2001) that reform teaching is scarce.

Typical U-M class

Members of the research team observed U-M math classes and took detailed field notes. We observed 3 Pre-Calculus classes (all in the first semester), 14 Calc I classes (9 in the first semester and 5 in the second semester), and 6 Calc II classes (3 in the first semester and 3 in the second semester). Structural features of each observed class were first tabulated, including the composition (numbers and gender) of each class, the arrangement of tables and chairs, the start/end time of the class, and the attendance rate. Next, the field notes were used to create categories of classroom actions. These categories included whether or not the class started or ended late, when and for how long students worked in groups (and what they worked on while in groups), when and for how long an instructor lectured (and on old or new material), and when students were assessed. These categories were then used to go back over the field notes and code the sequence of actions in each class. Finally, the collection of coded action sequences were analyzed, in an attempt to see if a portrait of a "typical" U-M class could be generated.

[5] Results from these survey instruments are not reported in this paper. The surveys have provided us with complicated, conflicting, and sometimes confusing results that we are still trying to make sense of.

Although there was some variation in what classes looked like in each course, our analysis indicated that there was a great deal of uniformity among the classes we observed. With the caveat that we only observed a small fraction of the Pre-Calculus and Calculus classes at U-M in 1999-2000, we were able to create a composite picture of what our participants typically experienced in their math classes.

Classes typically lasted 80 minutes (90 minutes minus a 10-minute passing period). In the classes we observed, males instructors outnumbered female instructors by a factor of 2, while the student gender balance was about half female and half male[6]. The 80 minutes of class time typically consisted of the following activities (see Table 1 for average number of minutes on each activity for each class): a series of announcements and collecting/passing out of work; reviewing of the homework/quiz/exam problems by the instructor at the blackboard; group work to practice material covered in a previous class, for an upcoming exam, or new material; and a lecture on new material.

(Insert Table 1 here)

It should be noted that the "group work" that we observed in most classes was not very collaborative; it can only be called "group work" because students happened to be sitting in tables of 4 and thus were in groups. In the typical case, at some point in the class, the instructor put a problem or two on the board and asked students to work in groups to solve the problem(s). The instructor circulated around the classroom and gave assistance to students needing help. During this "group work" phase of the class, we found that students rarely worked in groups. Each student typically worked on the problems individually, occasionally asking the person sitting next to him/her for help. Instructors typically did not do or say anything to indicate dissatisfaction with this mode of doing "group work". Occasionally we observed an instructor who reminded students that the point of group work was for them to work together and talk to each other, but this was rare. It would perhaps be more accurate to refer to this component of each class as "time spent individually practicing newly learned concepts or techniques while sitting at tables of 4", as opposed to "group work".

Also, note that the averages in Table 1 do not include "atypical" classes, such as when a quiz was given (4 of the classes we observed: three 115 classes and one 116 class) or when the instructor devoted the full class to reviewing for an upcoming assessment (3 of the classes we observed: one each for 105, 115, and 116). Average time for each activity type for these atypical classes is given in Table 2.

(Insert Table 2 here)

Other than the presence of "group work", the sequence of activities in Tables 1 and 2 indicate that the typical U-M math class does not utilize a lot of reform practices. While we are certainly aware of the differences often found between enacted and intended curricula (Smith & Berk, 2001), we were surprised by the extent of the similarities between the typical U-M class and high school Calculus classes which use more traditional curricula.

Published support documents provided to instructors by the mathematics department and discussed in instructor training describe much more of a reform teaching environment that what we typically saw in our classroom observations. The departmental instructor's guide for introductory classes (Shure, Brown, & Black, 1999) indicates that students should be "encouraged to experiment and conjecture, to describe and discuss" (p. 5) by working together,

[6] In PreCalc, females outnumbered males 2 to 1; in Calc I, the gender ratio was balanced; and in Calc II, males outnumbered females 2 to 1.

writing, and solving real-world problems. Instructors are asked to reduce the amount of time spent lecturing, to give up some of the control of the classroom flow, and to work hard to listen to students' responses and questions. More detailed sections of the instructor's guide are devoted to using cooperative groups in the classroom, different kinds of classroom activities, and questioning techniques. In general, we found the 40-page instructor's guide to be an excellent resource and one which describes a reform teaching environment much like we expected to see in U-M Calculus classes. However, our observations indicate that the department has had difficulty affecting change in GSI's teaching practices (more on this below).

We believe that the primary explanation for the lack of reform teaching practices in the classes that we observed is that most U-M PreCalculus and Calculus classes are taught by graduate student instructors [GSIs] who have very limited experience with teaching. For most, teaching at U-M is their first experience in the classroom; comfort level is often very low, especially at the beginning of each course. The mathematics department has in place a number of programs to support the development of inexperienced instructors, including a required one-week training session at the beginning of the school year, an extensive and detailed instructors' handbook (Shure, Brown, & Black, 1999), mid-term teaching evaluations conducted by a teaching and learning research center on campus, and sporadic classroom observations by course supervisors. However, these introductory courses have so many sections (and thus a large instructional staff), meaning that it is not usual for the quality of instruction to vary quite widely.

GSI's inexperience with teaching also brings with it a lack of awareness of reform teaching practices. A majority of instructors attended high schools in foreign countries and so have no experience with US mathematics reform efforts. Those who attended high school in the US did so at least 5 years ago and (as eventual math majors) were likely to be placed in advanced track courses; these two characteristics both reduce the likelihood that they would have experienced reform teaching. Given this lack of familiarity with reform practices, it is not surprising that a one-week training session would fail to affect major changes in these instructors' pedagogy.

Our conversations with U-M Calculus instructors indicate that many, especially those who are inexperienced, tend to offload their obligation to use reform teaching practices on to the group homework assignments. As mentioned above, group homework is a required part of the course. Students are expected to solve a collection of very challenging problems every week; collaboration, group discussion, and writing are integral to these assignments. Many instructors feel that the existence of group homework, along with the regular use of "group work" in class, satisfies the department's mandate to teach in a more reform manner. We found that if one were to remove the group homework assignments and the minutes of in-class "group work" from a typical U-M class, one would be left with a very traditional high school Calculus class, despite the use of a reform Calculus textbook.

"Flagging" potential cases of mathematical transition

With this picture of U-M mathematics classes in mind, we now turn to an examination of the students in our Year 1 sample. We seek to answer our research questions by looking closely at students' experiences, with an eye to identifying students who did and did not experience a mathematical transition. Once we have made this determination, we hope to learn more about the transition experience by uncovering patterns or trends in those students who did or did not feel the effects of the curricular shift.

We use four lenses in which to analyze students' experience. Each lens gives us a set of criteria in which to "flag" those students who appear to be having a noteworthy or difficult experience in their math classes at U-M -- in other words, those students for whom math class represented a 'bump in the road' in their first year of college. The collection of all analytical 4 lenses' perspectives allows us to determine what extent students' bumps in the road were the result of the curricular shift from traditional to reform as opposed to other, more general transitional issues. The four lenses that we use are: (a) student achievement or grades, (b) students' perceptions of differences between high school and college math, (c) dispositions toward mathematics, and (d) approaches toward learning of mathematics. In the sections that follow, we describe more what we mean by each lens, how we conducted the analysis for each lens, and which students can be flagged for a final transition analysis based on each lens. (For more detail on our methods and analytical framework, see Smith & Berk, 2001).

Student achievement. In this section, we look closely at students' grades in their math and other classes in high school and at U-M, with an eye toward flagging those students whose grades indicate a bump in the road which may be related to the curricular shift in mathematics.

The most obvious pattern which emerges from an analysis of students' grades is that almost all of the students' overall GPAs dropped in their first semester at the University of Michigan (see Table 3). Participants' mean first semester GPA was 3.17, a drop of 0.67 points from the high school mean GPA of 3.84. Individually, seventeen of the 19 students' GPAs dropped. The largest drops were 1.75 (FD) and 1.5 (BL) points. The only two students whose GPAs rose had very small rises: LS (a rise of 0.2) and KK (a rise of 0.11). This drop in students' grades between high school and college is not unexpected, particularly given that many U-M students enter college with very high GPAs.

(Insert Table 3 here)

A similar drop in grades shows up in students' achievement in mathematics classes (see Table 3). Students' mean GPA for their senior year of high school math was 3.43. The mean GPA for participants' first semester of U-M math was 2.98, or a drop of 0.60 points from the high school mean. Eleven of the 19 students had lower grades in their first semester U-M math class as compared to high school 12th grade math (largest drop was 2.0 points by DD and FD). Four students' U-M grades were the same as their 12th grade math grades, and 3 students' U-M grades were higher (highest rise was BD, 1.0 points). (One student, CA, dropped her first semester math course.)

This drop continued for those students who took a second semester of mathematics (13 of the 19 students; see Table 3). For the 11 of the 13 students who reported grades to us[7] (mean first term math GPA for these 11 students was 3.19; mean high school math GPA was 3.56), the mean second term math GPA was 2.74, an 0.45 point drop from the first term (and a 0.82 point drop from high school). Seven of the 11 students' math grades dropped (largest drop: 1.3 points by PJ and VJ). Two students' grades stayed the same as in the first term, and 2 students' grades rose (highest rise: 0.7 points for MT and DD).

This result, that students' grades were generally lower in college in all subjects, is not surprising. At a school such as U-M, incoming first-years have very high GPAs. Grades are likely to drop because of (a) a ceiling effect, where there is no room for grades to move in any direction except for down, and (b) the fact that college courses are generally considered to be more demanding than high school courses, in all subjects. However, our interest is in the role of

[7] Two students did not report their grades in the second semester.

the mathematical curricular shift in the grade drop in math classes. Determining whether such an effect exists requires considering how individual students performed in their mathematics classes relative to their overall GPA. In particular, a student whose overall GPA dropped but whose math grade rose presents a very different picture than a student with a similar drop in overall GPA and a corresponding and equal drop in math grade. For example, consider the cases of JC and MT (see Table 3). Both JC and MT did very well in high school, both in math classes (4.0 for both) and in their overall GPA (4.0 and 3.9, respectively). And both struggled in their first semester of U-M math, with each earning a grade of 2.3, or a drop of 1.7 and 1.6 points, respectively. However, JC's overall GPA fell to 2.6, while MT's GPA only dropped to 3.5. From our perspective, JC represents a case of someone who is likely experiencing a general development and transitional issue (see Smith & Berk, 2001); upon coming to college, all of his grades (including math) suffered a major drop. In contrast, in MT's case, something unusual seemed to be happening in math class; she did relatively well in all of her classes except for math. Cases such as MT, where grades indicate that something noteworthy may have been happening around her math class, are worth further investigation. In cases such as MT, there is a mismatch between the pattern of achievement in overall GPA and in mathematics; this mismatch is the criterion that we will use in this lens to indicate which students shall be flagged.

To what extent are cases such as MT prevalent in the Year 1 sample? Figure 1 gives a breakdown of how students' math grades between high school and college looked as compared to their overall grades: we have identified 3 different types of mismatches (indicated in bold).

(Insert Figure 1 here)

The first category of mismatch are students such as MT: those whose grades in all classes dropped, but whose grade in math class dropped even more. Of the 11 students whose math grade went down significantly in their first semester at U-M (a)[8], all 11 had GPAs which dropped as well (b). Eight of these 11 had overall GPAs which dropped as much or more than their math grade. Only 3 students, DJ, MT, and DD (c), had especially noteworthy drops in their math grades -- more than the drop in their GPA for all of their classes. For these three students, something unusual appears to be happening in math; they are worth flagging for further investigation.

A mismatch can also occur in other direction, where a students' grade in math class drop less (or even go up) while overall GPA goes down. For 2 students (BL and TM, (d)) while both math grades and overall GPAs went down, their math grades went down much less than their GPAs: BL's overall GPA went down from 3.6 to 2.1 from high school to college, but her math grade only went down from 2.5 to 1.7. The change in math grade was less (0.8 points) than the change in overall GPA (1.5 points) by at least half of a point, which, according to our criteria, suggests that BL should be flagged.

Something similar is happening with the two students whose math grades went up from high school to college while their GPAs went down (BD and VJ; (e)). For example, BD's overall GPA dropped from 3.8 to 3.1, while his math grade rose from a 3.0 to a 4.0. VJ's case is similar.

Thus, based on first semester grades, these seven students (MT, DJ, DD, BL, TM, BD, VJ) experienced a noteworthy change in their grades, where noteworthy refers to a relative change in math grade which was different than the rise or fall in overall GPA. In other words,

[8] Letters refer to where this number can be found on the figure currently referred to. For example, the (a) indicated here can be found in Figure 1, and it points to the 11 students being referred to in this sentence.

such students' grades show a mismatch between change patterns in math grades as compared to overall GPA and should be flagged.

Looking at students' second semester grades, several other students emerge as having noteworthy relative grade changes (see Figure 2). Of the 11 project participants who took a second semester of mathematics[9], 6 experienced a grade drop in math in the second term as compared to the first term (a). Of these 6, two students (VJ and CM, (b)) saw a rise in their overall GPA from 1st to 2nd term, which qualifies as a mismatch. Of the 4 students (c) whose overall GPA and math grades both dropped, 3 (BD, PJ, BM) had GPAs which dropped less than their math grade (d). Two more students can be flagged: LS, whose math grade in the second term was the same as it was in the first term while her GPA dropped (e), and DD, whose math grade went up in the second semester while his GPA remained the same (f). In total, the second semester's grades point to 7 students whose pattern of achievement can be classified as a mismatch. Of these 7 2nd term mismatches, 3 were already flagged from the first term (DD, VJ, and BD).

(Insert Figure 2 here)

Table 4 summarizes the results of using the lens of student achievement to identify students who experienced a bump in the road in their first year of college. Of the 19 students, 11 (7 from the first term, 4 additional in the second term) were flagged; Table 3 categorizes these students based on how they were flagged. The students flagged in Table 4 will be examined in more depth in later sections, after the remaining lens using in flagging students have been discussed.

(Insert Table 4 here)

Perceptions of difference. A second analytical lens that we use to examine students' experiences in mathematics at U-M is students' perceptions of the differences between high school and collegiate mathematics (see Smith & Berk, 2001, for a more detailed discussion of this lens). Recall that students were repeatedly asked to comment on what they perceived as different between high school and college math. Students were asked about this issue both with broad, open-ended questions, such as, "What did you find different between high school math and Calc I?", and more pointed questions, such as, "Was there anything about the instruction that you found different between high school and college math?". Students were also asked to confirm observations from prior interviews; for example, "In the first interview, you said that you felt that there was no difference between the instruction you received in high school and college math. Do you still feel that this is true?".

Students' responses to questions such as these were analyzed in the following manner. First, students' interviews and journals were transcribed. Next, a list of possible differences was generated in brainstorming sessions with project staff from all four data collection sites. In these brainstorming session, we identified features of students' mathematical experience that were mentioned as different by any student at any site during any interview or journal. This list was developed and refined multiple times over a period of several months. Ultimately, the list contained approximately 40 dimensions of difference, organized into the following five categories: curricular differences; differences related to teachers and teaching; differences related to site policy; differences emerging from the interaction of curriculum and teaching; and differences emerging from the interaction of curriculum, teaching, and site policy. (See Smith & Berk, 2001, for a more detailed discussion of these dimensions of differences.) The list of

[9] And participated in the project; recall that 2 students who continued in math dropped out of the project.

possible differences is shown in Table 5. Once this list and framework for differences was formalized, it was used to code interviews and journals at each individual site. At the U-M site, each kind of difference was given a alphanumeric code, and two independent coders went through all interviews and journals, coding for differences noted. In addition to coding for the mention of particular differences, the coders also indicated whether the difference was mentioned as a major or minor impact on the students' mathematical experience (and whether it was mentioned in a positive or negative light). The two coders subsequently met to resolve all disagreements.

(Insert Table 5 here)

This analysis provided us with two types of information. First, we identified which students noticed significant differences between high school and college math, where significant will be defined more specifically below. Doing so served as another means to flag students who experienced a bump in the road during their first year of college, as we assume that students were more likely to notice significant differences between high school and college math when these differences played an important role in their U-M math experience. Second, we identified which differences were noticed by a majority of students, and whether each difference had a major or minor, positive or negative impact on students' experiences in math. This second type of finding relates to our larger goal of understanding what specific features of curriculum students' notice as they experience curricular shifts (Star, Herbel-Eisenmann, & Smith, 2000). We present these two types of results separately.

Flagging students according to significant perceived differences. The fact that students had something to say when asked to describe what was different between high school and college math is not particularly surprising, given that they were asked about the issue specifically and repeatedly in interviews. All 19 students felt that some aspect of U-M math was different from high school math. In fact, when asked whether they would classify U-M math, as compared to high school math, to be "not at all different" from each other, "somewhat different", or "very different" (this question was asked during the first interview, which was about 6 weeks after the start of their first term math course), 7 students said "very different" and the remaining 12 said "somewhat different". In addition, of the 40 possible dimensions of difference that were part of our coding scheme (see Table 5), 37 differences were mentioned by at least one student on at least one occasion. In other words, when prompted to do so, students clearly had a lot to say about what was different (more on this below). Therefore, merely mentioning a difference when asked was not an indication of a possible bump in the road.

Thus our analysis sought to move beyond the mere mention of differences to capture instances where differences were significant -- that is, differences that were particularly meaningful, important, troublesome, or frustrating to students. We decided that if a student repeatedly mentioned particular dimensions of differences and indicated that these difference had a large impact on his/her experience in math, the student should be flagged for further investigation. More specifically, we decided that significant differences occurred when a student (a) repeatedly mentioned the same difference in at least 3 different settings (3 interviews, or 2 interviews plus journals), and (b) gave particular emphasis or attributed particular impact, either positive or negative, to the difference. Impact was assessed by the coders on a 5 point scale, where "1" meant a major, positive impact, "2" meant a minor, positive impact, "3" meant a neutral impact, "4" meant a minor, negative impact, and "5" meant a major, negative impact. "Particular emphasis" for a mentioned difference was operationalized as the presence of two or more minor impact codes (2 or 4) or 1 major impact code (1 or 5). We judged that the combination of repeated mention and emphatic mention would separate those students who were merely responding to our "What's different?" query from those for whom particular differences had important impacts on their experiences in math.

Table 6 shows the results of this analysis. On average, the 19 students made repeated or emphatic mention of 4.1 dimensions of difference[10]. The range of mentioned differences went from 0 (five students did not have any dimensions of difference which qualified as repeated or emphasized) to 9 (MT and DD). As a way to check the validity of our coding scheme, we compared the number of dimensions of difference mentioned by those who responded "very different" versus "somewhat different" (in the interview question which asked students to classify the difference as "very different", "somewhat different", or "not at all different"; recall that no one responded "not at all different"). On average, those responding "somewhat different" made repeated or emphasized mention of 3.4 dimensions, while those responding "very different" mentioned 5.1 dimensions. This difference was significant, $p<.10$.

(Insert Table 6 here)

As illustrated in Table 6, a student was flagged as having experienced a bump in the road, according to this particular analytical lens, if he/she repeatedly or with emphasis mentioned one or more dimensions of difference. Seventeen of the 19 students met this criteria[11]; these students will be discussed in more depth below, after the remaining two lens have been presented.

Differences noted. In addition to indicating which students may have experienced a bump in the road, the "differences" analytical lens also allows us to determine which features of U-M mathematics were most prominently different to students as compared to their high school math experience. As we discuss in prior work (Star, Herbel-Eisenmann, and Smith, 2000; see also Smith & Berk, 2001), curriculum designers and mathematics educators typically have well-articulated opinions on how traditional and reform curricula differ. But the issue of whether or not students notice anything as different has not been adequately explored. Tables 7 and 8 show the results of our analysis of what students did or did not notice as different between high school and college mathematics. Table 7 shows which dimensions of difference were most noticed by students, while Table 8 shows what students either did not notice or felt was not different. In both tables, these differences are organized by the 5 categories of differences in our analytical framework: curriculum (C), teaching/teachers (T), site policy (SP), the interaction between curriculum and teaching (C x T), and the interaction between curriculum, teaching, and site policy (C x T x SP). To make it into Table 7, recall that a difference had to be mentioned repeatedly and with emphasis; the mere mention of a difference, which occurred much more frequently, was not sufficient.

(Insert Table 7 here)

The most frequently mentioned and emphasized difference concerned the requirement to verbally or in writing explain one's solution steps while problem-solving (referred to by students as having to provide "explanations"), which is mandated by the department, the curriculum, and each individual instructor. According to the U-M math department's instructors guide, "students learn by writing. Writing forces students to organize their ideas and experience" (Shure, Brown,

[10] For the remainder of this section, when we say "differences mentioned", we mean differences that were repeatedly and emphatically mentioned. Recall that this is a small subset of all differences mentioned.

[11] At the time of this writing, we have completely analyzed all Year 1 data, with the exception of 2 interviews which have yet to be transcribed. It is perhaps no coincidence that the two missing interviews are from (ST7) and (ST14), who are the two students who (so far) do not met our criteria for having mentioned any significant differences between high school and college (see Table 6).

& Black, 1999, p.5). In addition, by site policy, students were expected to write several sentences explaining exactly what steps were taken in solving particular problems on group homework and exams; individual instructors were responsible for communicating these expectations to students and also for grading students' explanations. Students often felt that this writing requirement was tedious, unnecessary, and inconsistently implemented. Many did not see the benefit in being required to do such writing, and, according to students, instructors rarely talked explicitly about why this requirement was on the books. None of our Year 1 sample had experience writing explanations in high school, and, for many, this feature of U-M math was the most salient (and first mentioned) difference that came up in our interviews (see BL quote about explanations, Table 7). Some students recognized the benefits of writing; PJ commented that writing forced him to focus on "deep understanding of the concepts" (PJ, 10/26/99 Interview) rather than on merely calculating answers. In fact, even students who expressed strong dislike for having to write explanations, such as BD, saw the value of this requirement: "If you have to explain something to another person or you're in a work environment where you have to show a colleague how to do something, you have to be able to get across without just putting numbers on paper" (BD, 10/8/99 Interview).

Providing explanations to accompany problem solutions was an integral part of the group homework experience, which was the second most mentioned and emphasized difference between high school and college math. Recall that students were placed by their instructor into groups of 4 and were assigned a weekly problem set to be done in groups. It was expected that each individual would look over the problem set on his/her own, and then the group would convene (outside of class time) to discuss the problems and to write up a formal solution. Students in each group were to assume roles within their group ("scribe", "reporter", "clarifier", and "manager"), and these roles were supposed to rotate among group members week to week. None of the students in our sample had participated in formal group work in high school, and almost all made mention of this difference. While it was common in high school for informal groups of friends to complete problem sets and to study together for exams, formal group problem-solving with classmates who were not friends was a new experience. Students had difficulty finding common times to meet, and, as a result, often divided up the homework problems among the group to be done individually. Despite the many complaints about group work, particularly about the failure of groups to actually work together (as SB said, "My team homework is the exact antithesis of a team," 2/10/00 Journal; see Table 7), students did notice and comment on the benefits of solving problems in groups. According to MM, "So I think the group study homework sessions helped a lot because then you had 4 guys who were in the same or 3 other guys in the same situation as me. Not really knowing and just going on minimal you know what you can pick up in the book and each person can pick up a different type of concept and together we could make one person who actually knew what they were doing. It helped out" (3/26/00 Interview).

Another dimension of difference which inspired passionate statements from students was the quality of instruction in U-M Calculus. While many students made flattering comments about their high school teachers, opinions were more negative about U-M instructors. Many students perceived that their Calculus instructor did not particularly enjoy teaching (and perhaps would not choose to do it were it not required by the department or by financial necessity). Students commented that their instructor did not appear to care about helping students understand the content; as CA said, "I didn't hate math this much until I had him [my GSI]. ... because like last year my math teacher you know tried to help me around test problems and see different ways I could go about it. And he [my GSI] just said well you need tutoring. but that wasn't it because I know it and the GSI said you need tutoring, that's it. I don't have this problem. So he didn't help me at all he made me feel worse"(10/18/99 Interview). In addition, students frequently commented that an instructor was difficult to understand due to a foreign accent.

As a result of these difficulties with instructors, many students commented that it was necessary to do more work on their own. Whereas in high school a teacher or parent would insure that homework was completed or that students understood the material, at U-M this responsibility feel onto individual students. As BE said,

> It's a lot, I think it's a lot different than high school because um, you kind of teach yourself. Well in the beginning it was easy because it all was review. Now, it's getting harder so it's um, you got to study yourself and you can't always trust the GSI to like teach you because he's not a real great teacher. Because he knows everything and we just really, a lot of us don't know it, so he kind of goes through it kind of quick and um you kind of got to teach yourself. (10/21/99 Interview)

Although not as frequently mentioned as the differences mentioned above, students did notice a few things specific to the curriculum that differed from high school. In particular, almost all students commented on a difference in the typical problems done in U-M Calculus. While in high school students tended to do a lot of 'pure' symbol manipulation problems, such as calculating derivatives and integrals, at U-M the focus was much more on application or story problems. Particularly in the group homework assignments but also prevalent on exams, most problems were very wordy, situated the relevant mathematics in the context of a real-world example, and required some initial sense-making before one could launch into the use of mathematical procedures. For one student (SB) who had some experience with the CPMP reform curriculum in the beginning of high school, the predominance of word problems reminded her a lot of Core (SB, 10/8/99 Interview). Students generally saw the usefulness of doing lots of application problems; even JC, who had an especially negative attitude about a lot of his U-M math experience, was able to see the point of doing story problems: "The problems, they kind of force you to, apply them to real life situations. Like, the philosophy of the class for everyone when we went in there, was they wanted a math, cookbook math, for practical use, but you have to learn that the cookbook math and then apply it after that. Instead of just, trying to do both at the same time" (9/30/99 Interview).

Differences not noted. Table 8 shows the remainder of the differences in our coding scheme and which ones were mentioned (but not repeatedly or with emphasis) by students. In some cases, a mention by a student of an item in Table 8 was to report a difference, but in other cases, a mention (particularly since it was not repeated or emphasized) was to report that there was no difference between high school and college on a particular dimension.

(Insert Table 8 here)

One interesting finding in Table 8 is that students tended not to notice a lot of curricular differences between high school and college math. Other than the change in the typical problems as discussed above, students did not notice significant differences in the text used or in the topics covered (recall that almost all of our sample took AP Calculus in high school). To some in the field of mathematics education, it may seem obvious that the traditional Calculus curricula differs in some significant respects from the Harvard consortium materials; to students, however, no real difference (other than the increase in story problems) was apparent. Other "reform" teaching practices, such as the focus on multiple solutions to problems and different kinds of questions posed by the teacher, were also not noticed by students. Students did comment on the faster pace of college math classes, but this difference was expected and thus not particularly salient.

This analysis of what students did and did not notice as they moved from a traditional to a reform curricula at U-M will be combined with similar analyses at the 3 other sites, and we look forward to making more general claims about students' perceptions of curricular transitions in the near future.

Flagging students according to changes in approach to learning mathematics. In addition to looking closely at whether students' achievement in mathematics or their perceptions of difference between high school and college math indicate a possible bump in the road, we also used interviews and journals to determine whether students experienced a change in their learning approach in mathematics. By learning approach, we mean students' ways of going about the work of their math course -- what strategies or actions they took (or recognized that they needed to take) in order to be successful in math. Strategies can be things that a student does both in and out of math class, including such things as going to extra help sessions or office hours more frequently, hiring a tutor, reading the textbook more regularly, and taking notes in class. We define a significant change in a student's approach to learning mathematics to be when the student begins to use new learning strategies, lays aside old learning strategies, or uses old strategies in new and non-trivial ways (see Smith & Berk, 2001, for more details). When a student indicates possible changes in strategies but only in a very vague way, we did not consider this to be a significant change in learning approach. For example, if a student noted that that college math is harder so it became necessary to work harder, but he/she did not indicate any specific strategies that were done which constituted "working harder", we did not flag this as a change in learning approach.

Coding for strategy changes was done by a single coder, who read through all interviews and journals for each student, one at a time, and then made a determination of yes (indicating significant change in learning approach) or no. The coder wrote a paragraph giving reasons for his decision and also provided supporting evidence for his decision in the form of quotes from interviews and journals.

This analysis indicated that 5 of the 19 students experienced a significant change in learning approach in mathematics during their first year at U-M (see Table 9). Each of these five students realized that success in math would not come as easily as it did in high school, and that specific actions needed to be taken in order to try to be a better student. Merely "working harder" was not enough; each of these students articulated specific compensatory strategies that had been implemented upon experiencing difficulty in college math.

(Insert Table 9 here)

For BE, these specific actions began with the realization that he would have to teach himself -- that merely showing up to class and listening to his GSI would not be sufficient for him to master the material. When asked what he felt he must do in order to be successful in college math, he said, "Well number one you have to do your homework and keep up with it. Don't save it all for one night or like do it once a week. You got to do it after, do it every time after lecture. Always go to lecture, I'd say. And if you have questions get help from somebody else. I know like a lot of times towards the end of the semester I'd get help from a friend which always was a good idea to get help from somebody else. Definitely do your homework, I guess that's probably it, do your homework, read the book, go to lecture and ask questions" (12/16/99 Interview). When asked if these same strategies were what one needed to use in order to be successful in high school math, he replied,

> It probably is, but in high school math I probably, yeah I mean I'd say in any course you have to like keep up with stuff and but in like high school math I really didn't really need to read the book because the teacher's you could go to class 6 times or 5 times a week and the teacher would do go through everything really in depth and show you how to do it 100 times and stuff whereas in college you're doing a years worth of work in high school in a semester. And you meet three times a week. So I mean I think its' a lot different there's not near as much class time and class so and you go a lot faster so I think in high school it's really not as important, I mean it should be but it really isn't as important to really keep up and stuff. ... it's like if you don't it's easier to catch up in high school because you know you got through stuff a chapter in two weeks compared to like a week in high school, I mean in college, or about like that you know. (BE, 12/16/99 Interview).

Another student, LS, found it necessary to hire a tutor in order to earn the grades that she wanted in math class. After getting her first test back and scoring lower than she deemed acceptable, LS hired a student to work with her regularly, and she continued to work with her tutor for the next 2 years of math. She said,

> I just started that about three weeks ago when I realized in math 105 that I'm not pulling an A, so there's something wrong there so I needed to have somebody explain it to me in my own words on my time and that just happened to be useful to have a tutor. ... I got my first exam ... I thought I'd ace the thing and I had got, and it was a 76 but with the curve it was like a, it was a B- or something and that's not acceptable for having already taken Calculus in high school and wanting to be an engineer. I need to excel in math that's not really going to work for me. ... So I hurried up and got a tutor real fast. I probably could have done without it but I just want to be sure (LS, 11/4/99 Interview).

LS also mentioned that keeping up daily with the material was important to being successful in college math, and that doing so was much more the student's responsibility than it was in high school: "In high school math you can wait for someone, the teacher wants you to grasp all these concepts. They will drill it to you, they care, they hound you to come after school. They will bend their schedules any which way to meet your schedule and it's just a lot different, you're responsible for it in college and in high school you can if you just want to depend on your teacher to make certain you learn it it's fine cause they will" (LS, 12/16/99 Interview).

Other specific strategies that students implemented when faced with difficulties in math class included going to extra help sessions and office hours regularly, studying with peers, reading the textbook more, and completing the practice exams (which were available online) prior to exams.

Among the students who were not flagged as having made significant changes to their learning approaches, some indicated, albeit in a vague way, that since their college math course was harder, they just had to work harder in order to succeed. For JC, the change from high school to college math was just a change in the routine of math class, and adjusting involved merely getting used to the new routine. He said, "I think it's just well the whole group work thing I've adjusted to. Still don't agree with it 100% but it I mean it comes easier now and we just run through it because we have a routine now and we run through the same routine now and the whole teaching thing, or the in-class discussion and everything that was never bad really so there's really no adjustment there and the homework is kind of just like the same homework as in high school, same format" (JC, 10/28/99 Interview). Other students explicitly mentioned that they were not doing anything differently in their learning approach as compared to high school, despite the differences that they may have noticed between high school and college math.

Whether or not students made a change in their approach to learning was related to changes in their grades. As will be discussed in more depth below, all 5 students who made a change in their learning strategies were also flagged using the achievement change lens.

Flagging students according to changes in disposition. A final lens through which we identify students who experienced a possible bump in the road is their disposition toward mathematics. Disposition encompasses students' attitudes about, interest in, motivation to succeed in, and enjoyment of mathematics. Disposition came up in interviews and journals as students commented on their experiences in mathematics, their short-term goals (e.g., taking more math courses) and long-term goals (e.g., career or major choice). Change in disposition was determined in the same manner as change in approach to learning; Table 10 shows the results of this analysis.

(Insert Table 10 here)

Eight of the 19 students experienced a significant change in their disposition toward mathematics. Five of these 8 students experienced a negative change in their disposition -- particularly a decrease in their enjoyment of math. For example, BL said that she loved math in high school: "I love math, like I always liked math, and I always like usually get it if I really work it" (BL, 10/6/99 Interview). Despite having some difficulty in high school Calculus, she entered U-M with a strong enjoyment of math. However, her attitude changed quickly, after only a few weeks in U-M math, she reported that, "I hate math here" (BL, 10/6/99 Interview).

Similarly, CA came to U-M with a mild dislike of math, but quickly developed a strong dislike for her U-M math course. With regard to math in high school, she said, " Um, I didn’t like it but I didn’t hate it as much as I do right now" (CA, 10/18/99 interview). However, she did indicate that she was planning on taking another semester of math: "Yeah, I figure I’m going to take 116 just because last year we went up through a lot of Calculus II. And I might as well, because I have the background in it, and the book" (CA, 10/18/99 interview). However, about midway through the first semester (and a few weeks after our interview with her), CA decided to drop her math course. She attributed her difficulties to her GSI, whom she hated: "I hate that man. I didn't hate math this much until I had him" (CA, 11/20/99 Interview). In fact, CA was planning on majoring in Pre-Med, but after her experience in math, she decided to change her major to something which did not require taking any math.

DJ had a similar experience in her fall semester. She loved math in high school, but she had a very difficult experience in U-M math in the fall. After completing her first term of math at U-M, DJ decided that she was not interested in taking any more math courses. She said,

> Well, I'm like looking where I want to go in the future and I just don't think math is going to be something I really need. I was thinking of going into pre-medicine but and that I mean after [first] semester I don't know math didn't seem very interesting to me. ... Before I loved math. Math was basically numbers and it's not like science where this can happen but this can also happen. It's just straightforward like 2 + 2 is four it's not five. You can't make it five. But now I get I don't know I guess I might've liked it more if I'd done better at it. ... like my [high school] teacher she explained it so I could understand it. I don't know. She was a pretty good teacher.... Sometimes I didn't always do the

> problems math in high school. But I still did fairly well. She explained it so I could understand it, she made, I guess because it's the whole high school thing it's different. She made sure we were up to date, on track. But like in college well [my GSI] just kind of teaches and after class that's it. You know you do it on your own. Like I guess I wasn't really motivated to do the problems. (DJ, 3/29/00 Interview)

In contrast, two students (BD and KK) reported increases in their enjoyment of math as a result of the U-M experience. In KK's case, although she thought that much of Calc II was review for her, she felt that the way the course was taught made her learn the material more thoroughly and deeply than she had in high school: "Like I feel like I'll come out of 116 knowing like knowing better the stuff that I would've you know like coming out of algebra in high school. I mean then I could kind of manipulate it but I don't think I really knew it. I think had I not kept using those algebra skills that I would've completely lost them. And even though I might not keep using my integration skills I don't think I'll lose them because I know where they came from and I know how they work so I should, I mean I think it's like riding a bike. I'd be able to go back and do it again." (KK, 12/13/99 Interview). Although she did not take any more math courses[12], she was very glad to have taken 116 and felt that she had learned a lot in the course..

Similarly, while BD initially had complaints about being required to provide written explanations (see above), he ultimately felt that doing so helped him to more fully understand concepts that he had previously studied in high school: "I think [having to write explanations is] useful because at first I didn't like it but more, now that I look back some of the concepts that I could use and do in high school, I understand now. Before I just knew that that was what you did, now I understand why you do it and I think that's what this process allows for" (BD, 1/19/00 Interview). BD recognized both the difficulty of having to provide written explanations and also how useful this process is to the development of understanding. As an engineering major, he was especially interested in understanding the 'why' behind the mathematical procedures he had learned in high school, and he felt that Calc I provided him with this knowledge.

Mathematical transitions.

The previous sections attempted to identify students who had possibly felt a bump in the road in their first year of U-M which was related to their experience in mathematics classes. Four independent analytical lenses were used to make this judgment: student achievement (whether a change in math grade between high school and college occurred, relative to any change that happened in overall grade point average); perceptions of differences (whether a student made repeated or emphatic mention of some differences noticed between high school and college math); disposition (whether a student's attitude toward mathematics changed significantly during the U-M year); and approach toward learning (whether a student's strategies for succeeding in math class changed significantly during the U-M year). For each student, a yes/no determination was made from each of the lenses, and the overall results of this flagging process are shown in Table 11.

(Insert Table 11 here)

One of our aims in this research is to identify whether students experienced a mathematical transition or not, and Table 11 allows us to make an initial attempt at this. We propose that students who are flagged with a 'yes' on one or fewer of these lenses did not

[12] Very few students go on to take Calc III, as it is not required for any majors at U-M other than mathematics and engineering.

experience a mathematical transitions; students who are flagged with a 'yes' on three or more of these lenses have had a mathematical transition, and those who received two flags need to be examined in more depth in order to make this determination (see Smith & Berk, 2001, for more details on this). Such an analysis indicates 2 students (GD and MM) who did not have a transition, 5 students who did (DJ, BD, BL, TM, and LS), and remaining 12 students who need to be examined individually to determine if they did or did not. The individual analysis of these 12 borderline cases by the project staff resulted in classifying 2 persons as 'yes' (DD, KK), 4 persons as 'no' (CA, FD, JC, BE), and the remaining 6 were left as 'maybe'. Thus, we were left with the following: 7 students who we believe did experience a mathematical transition, 6 students who we believe did not, and 6 students for which this determination was difficult to agree on. It is worth looking briefly at these three groups of students in more depth to see whether our formula for determining transitions has face validity. Doing so may also give some insight into why some students experience transitions while others do not.

The 'mathematical transitions' group. The 7 students who fall into the 'yes' category are those whose interviews and journals indicate that they experienced a bump in the road as a result of the curricular shift in their math course. This bump showed up in more than one of our analytical lenses but was particularly noteworthy in the achievement and differences lenses. Six of the 7 were flagged as having had a significant change in achievement in math class relative to the rest of their classes (everyone except KK); all 7 were flagged as having noticed (and commented on repeatedly and with emphasis) significant differences between high school and college math. Looking more carefully at these 7 students, it is clear that there are two different types of students who experienced mathematical transitions.

Four students (BD, DD, LS, and KK) had what could be called a positive transition. For these 4 students, it seems that the U-M math experience was transformative. Each of these four students noticed that U-M math was very different from high school (all four responded 'very different' when asked about this difference; see Table 6 above); in fact, DD and LS were among the students who repeated and emphatically mentioned the most number of differences (9 and 8, respectively). Group work and having to provide explanations inspired a particularly prominent and initially negative reactions among these four students.

However, at some point in their first year, each of these students became convinced that the U-M curriculum was actually quite beneficial to their learning. KK's and BD's comments in which each expressed a realization of the usefulness and importance of having to provide explanations (see previous section) illustrate improvement in each's mathematical disposition; KK and BD expressed that the belief that techniques of integration would now not be forgotten because the "why" had been learned, not just the "how". DD noted that group work was useful because, "it forces you to work with other people and to work through hard problems" (DD, 12/13/99 Interview). LS experienced the same kind of turn-around in her feelings about U-M Calculus, although for her it took a bit longer.[13]

In contrast, the remaining three students (DJ, BL, and TM) experienced a negative transition. Each of these three had a very difficult math experience, with drops in math grades and significant change in either disposition, learning strategies, or both. Both DJ and BL entered U-M enjoying math and finished their first semester hating it; TM struggled in all of her courses at U-M but had a particularly hard time in math. The strong and negative feelings that each of

[13] (ST18) continued to be a part of the project in Year 2, and we base this claim on interviews that were conducted (but have not yet been transcribed) in the fall of 2000.

these students had about group work, explanations, and about their GSI never subsided. BL and TM implemented significant changes in their approaches to learning; new strategies included getting regular extra help from peers and from math department help sessions, reading the textbook more, and completing the daily homework more regularly. But none of these new strategies seemed to help these three students improve their grades.

What distinguished these three students from others in our sample who may not have done well in math (e.g., CA, MT, FD, and JC; see below) is that each realized and commented that U-M courses were asking them to know the mathematics in a different way. BL commented that in her U-M math course, one must know not only what a concept or procedure is and but also be able to explain how to apply it -- "The level of understanding in college is much deeper," (BL, 12/15/99 Interview). Similarly, DJ reflected on a poor exam performance by complaining about the different kinds of problems that were typically asked in U-M classes: "I knew how to do that, but with all the story problem I didn't know to do that, like if they had given me a straight equation and said 'Do this.' I could have done it" (DJ, 3/29/00 Interview).

To summarize the two types of transitions group, what all 7 students have in common is a realization that U-M math required the development of a different kind of knowledge as compared to mathematics courses in high school. While in high school, one could do quite well by merely knowing how to execute procedures, in college one needed to know the material more deeply and conceptually. The course structures which students complained so much about -- group work and writing explanations -- were realized, to the transitions group, to ultimately be instrumental in the development of this different kind of knowledge. However, students who experienced a positive transition figured out how to succeed in math class after becoming cognizant of this fundamental shift, and thus their complaints about group work and explanations died down. In contrast, students who experienced a negative transition realized this fundamental difference in what they were expected to know but were unfortunately unable to adapt.

The 'no mathematical transitions' group. The 6 students who fall into the 'no transitions' group (CA, FD, JC, GD, MM, BE) are those who either did not experience a significant bump in the road upon coming to college or those whose bump seemed to be a result of general issues rather than those specific to their mathematics class. None of these six students experienced a significant change in their performance in math class as compared to the rest of their classes, and 5 of the six (everyone except BE) also failed to change their approach to learning in U-M math classes. As was the case above, these 6 students can be subdivided into 2 groups -- one group who experienced a general, non-mathematical transition and one group who did not experience a transition at all.

Three students (CA, JC, and FD) experienced a general transition upon coming to college. All three of these students had a negative change in their disposition toward math. However, there is evidence that their negativity was not exclusive to math but rather affected much of the rest of their first year experience. CA struggled in both math and chemistry, and in November of her first semester, dropped both courses and decided to change her major. JC and FD had 4.0 grade point averages from high school, but finished their first term with GPAs of 2.6 and 2.3, respectively (and with comparable drops in their math courses).

As mentioned above, what distinguished these students from others who did poorly in math (in addition to the more general drop in the rest of their grades) was their failure to take notice of fundamental shifts in what they were expected to know in U-M math as compared to high school math. All three of these students noticed differences (although few were repeated and emphasized; CA, FD, and JC mentioned 1, 1, and 4 differences, respectively); however, these differences were confined to structural features of the course (the GSI, the presence of group work, being required to provide explanations) and did not touch on more epistemological issues.

For example, CA was a bit unsure as to why she was having problems in math -- she thought she knew the material but somehow she struggled on tests: "Yeah, I was having problems taking the tests in the class. I knew all the material forwards, backwards, in my sleep I was showing other girls in my hall how to do it. But I would get to a test and I would forget absolutely everything. And it was driving me nuts. ... Like we were doing derivatives when I dropped. I can do derivatives forward, backwards, in Chinese. I was showing other people how to do them but I would get to a test and I would be like so what does that mean again" (CA, 11/20/99 Interview). In addition, CA had kept a very detailed and complete set of notes from her high school Calculus class, and she found that these notes did not help her complete the kinds of problems that she was asked to do in U-M math.

Similarly, JC commented repeatedly on having to provide written explanations, but he viewed this primarily as a tedious exercise rather than one which required him to know the material in a different, deeper way. "Explanations" were all about figuring out which words the grader wanted to see in a particular answer, and what the grader decided was "right" seemed quite arbitrary:

> But then the exams came around and ... if you didn't have like key words in your verbal explanations or if you didn't do something extremely specific then you got points marked off ... I know I felt really comfortable with the material and I can use derivatives or whatever on the exams pretty effectively but it's just like the questions are sometimes kind of vague so it's hard to know what angle to approach things from so that you're giving them what they want, you know what I mean? A lot of time you could get the right answer like the right numeric answer and you give all the stuff that they want but there would still be points taken off. I man and there's questions that ask you just for the answer but you have to explain, regardless. (JC, 12/16/99 Interview)

Later in this same interview, JC gave some indication that he had given thought to different ways in which one was expected to know the U-M Calculus material, but for him, these differences amounted to an increase in abstraction (less numbers and more variables, which he calls "conceptual") and an increase in explanations, rather than a need to know the mathematics in a different way: "I mean that you can get by on everything but the exams just doing like the cookbook math. The final exam was pretty much all conceptual; there really wasn't any numbers involved. It was either like algebraic or you had to explain what to do in order to find the answer" (JC, 12/16/99 Interview).

For FD, U-M math clearly required him to do more than merely calculate answers to derivative and integration problems (which he was able to do with relative ease). However, he was never particularly clear on what this 'something more' was or whether or not it was even related to mathematics. In high school, FD said that, "it was always before a sheet of problems just do the problems and you have to understand them and apply them and everything and [now 115] was just totally different and I didn't feel like I had, should I need to put so much time into you know doing it, cause I never had to put any time into it before. ... Like in high school it was just problems just do them, find the answer. Now it's related to other things you know" (FD, 3/26/00 Interview).

Although these three students (FD, JC, and CA) noticed many things to be different between high school and college, they did not experience a mathematical transition because they failed to notice substantial and epistemological differences between the math they had learned in high school and how they were expected to know similar material in college. In addition, each of these students experienced a more general transition (and a performance drop) in all of their courses, including math.

Another distinct group of students who did not experience a mathematical transition were those who coasted through U-M math without experiencing much difficulty. GD, MM , and BE took AB Calculus in high school and scored well on the AP exam (3, 4, and 5 respectively). All three students placed into Calc II and did well in this course (as well as the rest of their courses). None of the three experienced a disposition change, and only BE was classified as experiencing a change in his learning approach.[14] Although they did notice differences between high school and college math, none of these three students was particularly impacted by these differences. All commented that much of Calc II was review for them, and they had the advantage of being able to adjust to perceived differences within the context of familiar material.

The maybe's. Six students did not fall nicely into the transition or no transition groups. In other words, there was not a compelling reason to classify these students as either yes or no in terms of a transition. All noticed some dimensions of difference between high school and U-M; their grades and relative grade changes varied; their reactions to U-M math also varied. In our continuing analysis of the Year 1 data and with the addition of Year 2 data, we will return to these students again in an attempt to refine our categorizations.

Table 12 summarizes our conclusions about whether students did or did not experience a mathematical transition.

(Insert Table 12 here)

Discussion and conclusions

We set out to answer three research questions in this paper. First, what do students notice as different in the current mathematics experience, as compared to what was experienced one year ago? Our Year 1 data indicates that students noticed a lot that was different in U-M math as compared to high school. However, these differences tended to focus around issues of teaching, explanations, and group work, rather than on features of the curriculum. Second, what mathematical transitions do students experience? By looking at changes in students' achievement, disposition, and learning approach, as well as perceptions of difference, we proposed that 7 of the 19 students had a mathematical transition (4 positive, 3 negative) while 6 did not (and we were unsure of the remaining 6). Those who experienced a transition seemed to be the ones who realized that U-M math made a different set of expectations about what it meant to know and learn mathematics, while those who did not notice this fundamental shift were not as affected by it.

We find this result about mathematical transitions to be very interesting. Stated somewhat differently, our results indicate that students by and large did notice fundamental shifts in what it means to know and do mathematics as a result of moving from a traditional to a reform curriculum. The only students who failed to notice this shift were those who had a broad and

[14] This change in (ST8)'s approach to learning actually came about primarily in his second semester of U-M math, when he took Calc III, which is a traditionally taught course. Thus his strategy changes are not particularly related to the shift from traditional to reform curricula.

general decline in their academic work as they moved from high school to college and those who, by virtue of advanced preparation in high school, were able to coast through college math without having to seriously engage. For those students who noticed, adjusting to this shift required a change in learning strategies and/or disposition. However, for some students, particularly those whose training in mathematics was the weakest, even adjustments in learning strategies did not enable them to navigate their way through the altered set of mathematical expectations.

As for our third research question, which asks about the resources and actions that individuals and departments can undertake in order to support more successful transitions, we have some interesting leads but still much work to do. For example, the strategies employed by BL when she became cognizant of a fundamental shift in mathematical expectations in PreCalc, which included doing more homework, going to class everyday, reading the text, and doing the practice exams, were of only limited help to her, while these same strategies seem to be more helpful to other students in a similar plight. As another example, LS seemed to benefit greatly from the one-on-one help that having a tutor provided, yet clearly a private tutor is not a generally available solution. We believe that our results at other data collection sites, as well as the Year 2 data that we are currently collecting, will help us shed more light on this complex problem.

Another issue that merits further study is the categorization of the six students for whom we had difficulty making a definitive yes/no decision on the issue of mathematical transition. These students defied easy categorization: Were they successful? Did they experience a transition? As a group, they are a mixed bag. We hope that the patterns of successful and unsuccessful transitions that we will encountered at our other sites will help us to better understand these more complex and fuzzy cases.

With these promising beginnings from our Year 1 analysis comes a recognition of the challenges that we face in our continued research. We list several of these challenges. First, we realized this year that U-M draws upon a very successful and specialized population of high school students (typically, those with high GPAs, 4 years of mathematics, and lots of AP courses); we need to give thought to how the extremely successful math backgrounds of the students in our sample may affect both our results and our ability to integrate our findings with those of the other 3 research cites. Second, we have gained an appreciation for how difficult it is to track the full experience of a relatively large number of students. We will need to give thought to how we improve our data collection efforts so that we can find out as much as possible about U-M students' experiences in math classes and in college generally. Finally, we have found our job complicated by the limits on students' course taking in mathematics. Only about half of our original Year 1 sample of 19 students took mathematics for a full year. We may need to increase our sample size to adjust to this kind of attrition.

Our efforts in Year 1 have also raised a number of new questions that we hope to explore in more depth. First, we are interested in exploring the long-term effects of the Harvard Calculus program at U-M. For students who take mathematics courses beyond Calc II, the curriculum goes back to a more traditional one. Do students experience this curricular shift as another mathematical transitions? In what ways is this transition related to the one which occurred as students moved into the Harvard courses? Also, we are interested in learning more from the students who stopped taking math. How do their reflections on the experiences in U-M math change over time?

Second, as we continue to learn more about the transitions that students experience, we are interested in finding out how different groups may play roles in helping students navigate any difficulties that arise. How does a University, a department, a class, or a homework group contribute to students' negotiation of mathematical transitions? What resources offered by these different types of groups do students make use of, and how affective are these resources?

We have just begun to analyze the data from our second year of this endeavor, and we look forward to building upon these initial findings in the years ahead.

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Table 1
Average time (minutes) spent on various class activities in a typical class for each course

	PreCalc (n=2)	Calc I (n=10)	Calc II (n=4)
Start time	(on time)	(on time)	1 (late)
Announcements/Hand out papers	8	7	1
Review of homework (Instructor at board)	32	23	6
Group work to practice material	13	27	33
Lecture on new material	15	13	34
End time	9 (early)	6 (early)	(on time)

Table 2
Average time (minutes) spent on various class activities for atypical classes for each course

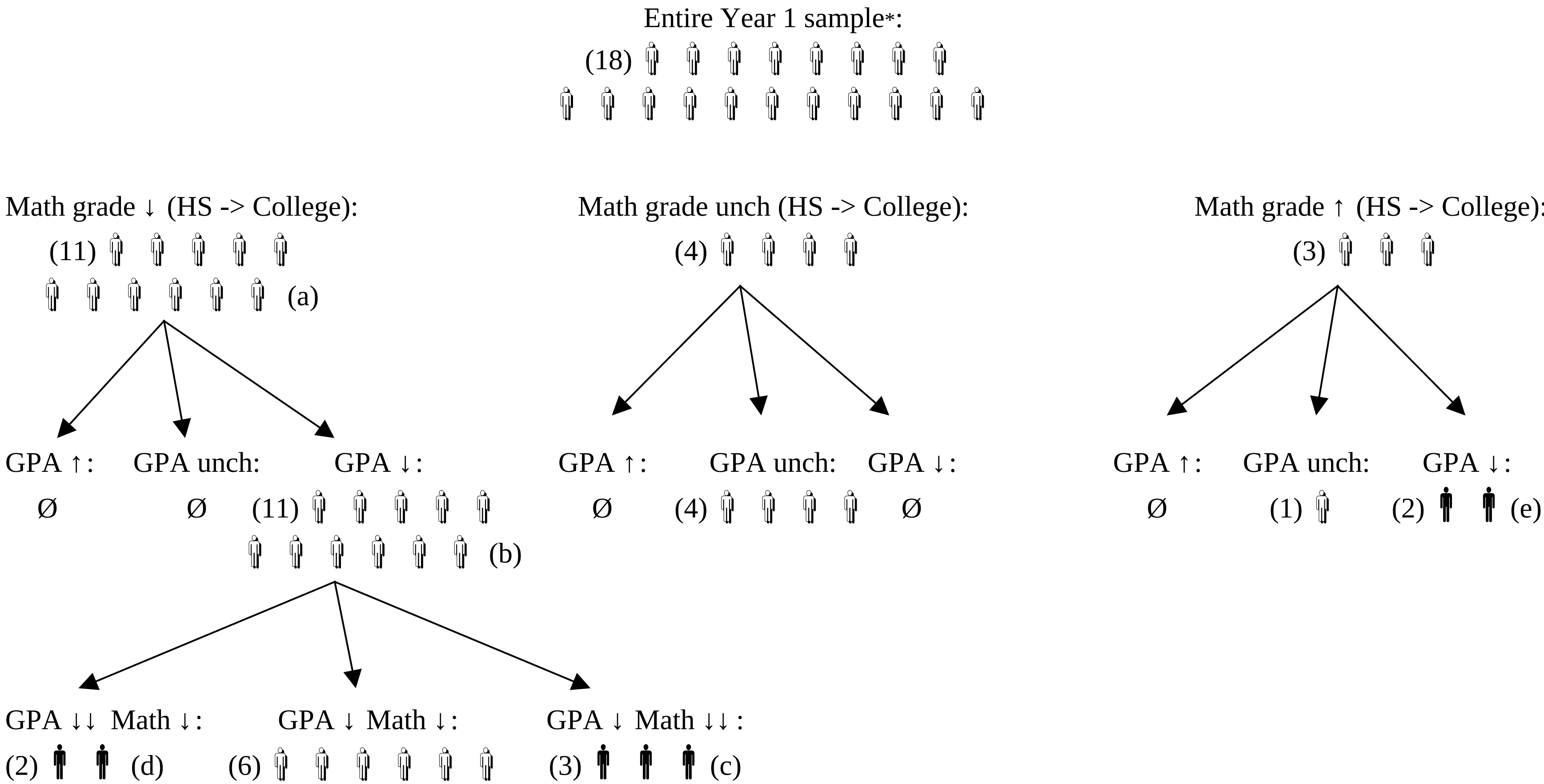
	Quiz Class (n=4)	Review Class (n=3)
Start time	1 (early)	
Announcements/Hand out papers	3	5
Quiz	30	
Review of homework/quiz problems (Instructor at board)	18	80
Lecture on new material	12	
Group work to practice new material	10	
End early	6	

Table 3
Year 1 Achievement Data, U-M site

	HS GPA	HS (12) Math grade	1st semester GPA	1st semester math grade	2nd semester GPA	2nd semester math grade
CA	3.5	0.7	3.3	nd	nd	nd
SB	3.8	3.7	3.7	3.7	3.0	3.3
JC	4.0	4.0	2.6	2.3	nd	nd
BD	3.8	3.0	3.1	4.0	2.7	2.7
DD	3.9	4.0	2.7	2.0	2.8	2.7
FD	4.1	4.0	2.3	2.0	nd	nd
GD	3.9	4.0	3.5	3.3	nd	nd
BE	4.0	4.0	3.7	3.7	3.8	3.3
DJ	4.0	4.0	3.2	2.7	nd	nd
PJ	4.0	4.0	3.9	4.0	3.2	2.7
VJ	3.8	2.5	2.7	3.3	2.8	2.0
KK	3.9	4.0	4.0	4.0	nd	nd
BL	3.6	2.5	2.1	1.7	nd	1.7
BM	3.9	4.0	3.8	4.0	3.1	2.7
CM	4.1	4.0	2.9	2.7	2.9	2.3
MM	4.0	4.0	3.6	3.3	nd	nd
TM	3.2	1.5	1.9	1.0	nd	nd
LS	3.7	3.5	3.9	3.7	2.8	3.7
MT	3.9	4.0	3.5	2.3	nd	3.0

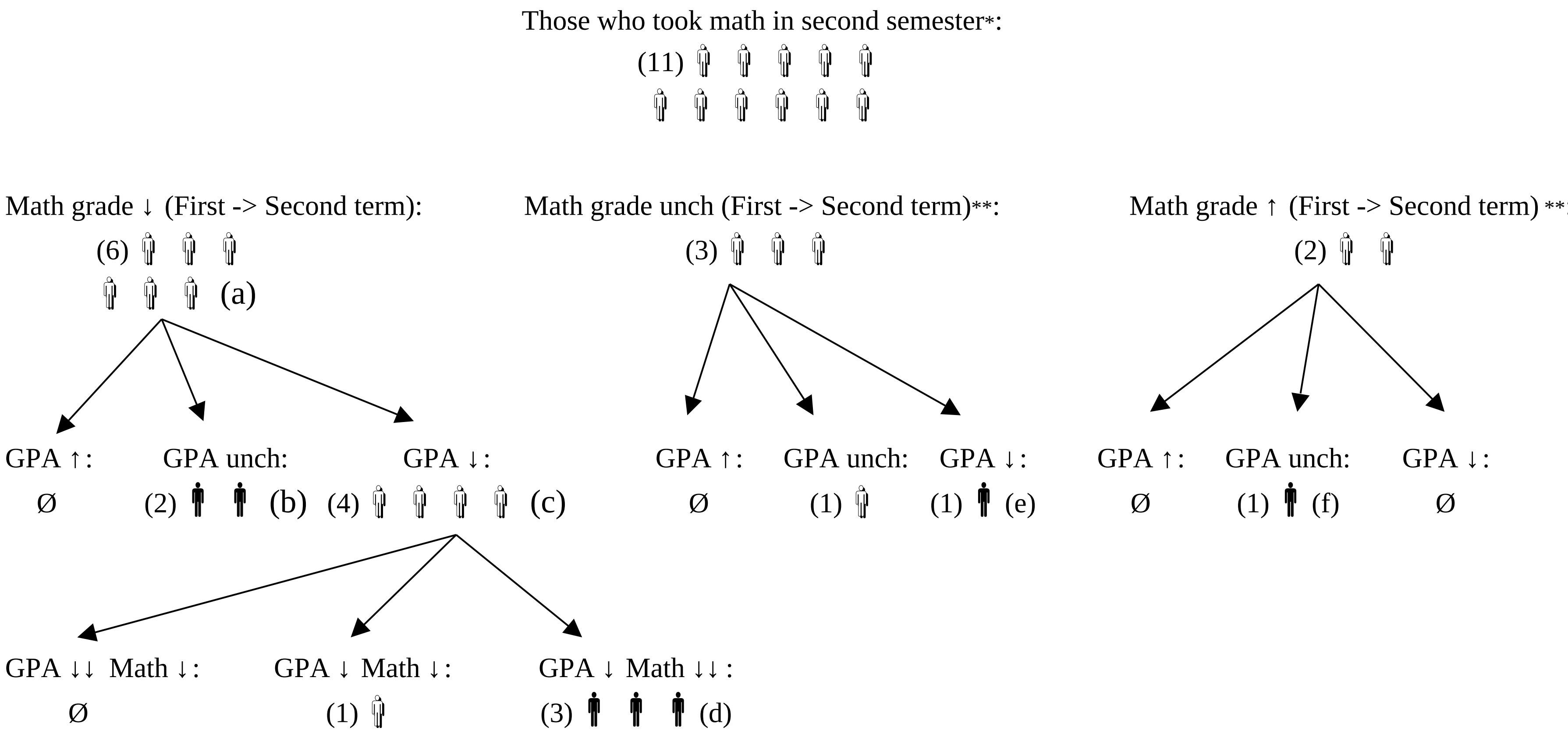
*nd = no data, because student did not take math, dropped the course, or were dropped from the project.

Figure 1
Breakdown of first semester math grades and overall GPA*



*Not including CA, who dropped math midway through her first semester. Characters which are darkened in ( ) represent flagged individuals.

Figure 2
Breakdown of second semester math grades and overall GPA*



* Characters in shadow ( ) represent flagged individuals.
**Second semester GPA not reported for one student in this category

Table 4
Summary of flagging based on achievement[15]

	Achievement flag?	**Details:**
CA		
SB		
JC		
BD	yes	Math ↑, GPA ↓ (1st term); Math ↓↓, GPA ↓ (2nd term)
DD	yes	Math ↓↓, GPA ↓ (1st term); Math ↑, GPA unch (2nd term)
FD		
GD		
BE		
DJ	yes	Math ↓↓, GPA ↓ (1st term)
PJ	yes	Math ↓↓, GPA ↓ (2nd term)
VJ	yes	Math ↑, GPA ↓ (1st term)
KK		
BL	yes	Math ↓, GPA ↓↓ (1st term)
BM	yes	Math ↓↓, GPA ↓ (2nd term)
CM	yes	Math ↓, GPA ↑ (1st term)
MM		
TM	yes	Math ↓, GPA ↓↓ (1st term)
LS	yes	Math unch, GPA ↓ (1st term)
MT	yes	Math ↓↓, GPA ↓ (1st term)

[15] Arrows (↑ and ↓) indicate change between high school and first term of college, or change between first term and second term of college. For example, an up arrow (↑) in the math grade in the first term indicates that a student's grade rose between high school and the first term of college. "Unch" designates a grade which did not change.

Table 5
Framework for noticed dimensions of difference[16]

Curriculum (C):
- surface characteristics of text (e.g. color; number, and size of volumes)
- typical problems (e.g. "story" or "book")
- direction provided for solving problems (e.g. explicit procedures vs. general description of solution processes)
- non-verbal representations of mathematical relationships (e.g. tables of values, graphs, equations, diagrams)
- content is different (e.g. geometry is different from algebra)
- presence of non-mathematical topics or problems (topics not typically thought of as "math")
- absence of expected or desired mathematical code as topics (e.g. algebraic manipulation)
- diversity of mathematical topics (over semester or year)
- short-term coherence/connection among topics (within chapter or unit)
- long-term coherence/connection among chapters or units (over semester or year)
- terms in text(s) difficult to understand
- passages in text(s) difficult to understand (descriptions, explanations, etc.)

Teachers/Teaching (T):
- relationship with teacher (accessibility, trust, care, sense of humor, contact outside of classroom)
- basic communication differences (student cannot hear, understand, or follow teacher's speech)
- classroom management (e.g. other students inattentiveness is distracting)
- activities in typical lessons differ
- sequence of activities differs
- duration or importance of activity differs
- way of conducting activity differs
- nature of questions posed by teacher (frequency, type)
- nature of student questions supported by teacher (frequency, type)
- pursuit (or lack of pursuit) of multiple solutions to problems
- size of groups (for group homework)
- constituency of groups (assigned or chosen by students)
- teacher's use of terms not used in the text
- pace of teaching (as determined by individual instructor, not Department)
- assessment (quizzes and tests, if designed by individual instructor, not department)
- have to do more work on own

Site Policy (SP):
- pace of presentation of material (if dictated by department)
- larger class size
- increased anonymity, particularly in lecture (e.g. "I don't know anyone in this class")
- final Department-wide exams do not represent "what we learned" in individual lecture/section

[16] The framework is Table 5 is what was used at the U-M site. Slightly different frameworks were used at the 4 project sites. Although all shared the 5-part framework of curriculum, teaching, site policy, and the two interaction categories, some of the specific details of the component dimensions varied slightly. We are still in the process of refining and finalizing this framework, and ultimately all sites will use a common list.

(Table 5, continued)

Curriculum x Teachers/Teaching (C x T):

- presence or absence of group homework
- disadvantages of group homework (to students)
- benefits of group homework (to students)
- typical homework assignments (number and difficulty of problems)
- support for developing a deeper understanding of mathematical ideas (more than previous curricula)
- other curriculum x teacher/teaching issue (list here)

Curriculum x Teachers/Teaching x Site Policy (C x T x SP):

- required to verbally or in writing explain solutions
- required to explicitly show all steps in solution

Table 6
Summary of flagging based on mentioned dimensions of difference

	Differences flag?	**# differences noted:**	**very or somewhat**
CA	yes	1	very
SB	yes	4	somewhat
JC	yes	4	somewhat
BD	yes	2	very
DD	yes	9	very
FD	yes	1	somewhat
GD		0	somewhat
BE	yes	6	somewhat
DJ	yes	1	somewhat
PJ	yes	6	somewhat
VJ	yes	2	somewhat
KK	yes	4	very
BL	yes	4	very
BM		0	somewhat
CM	yes	7	very
MM	yes	4	somewhat
TM	yes	4	somewhat
LS	yes	8	very
MT	yes	9	somewhat

Table 7
What students noticed as different in college math as compared to high school[17]

Category	Difference	Repeat & Emphasis	Mere Mention	Example quote
C	typical problems -- more "story"	5	16	"In high school we really didn't, I mean we did a few story problems but mostly it was just like you know problems you know. These, the ones that we do now, like even for individual are more like story problems and its just they're like different from what I used to do I guess." (CM, 11/4/99 Interview)
	direction provided for solving problems	3	12	"I mean I like, I kind of liked our high school class just because our teacher, as long as you got the right answer, he didn't care how you got it. Like, it seems like in this [U-M] class they really prefer ... you use the method they taught you and if you don't use that you're probably not getting any credit even if you do come up about getting the right answer." (PJ, 3/26/00 Interview)
	content is different	3	12	"I have been working on the practice tests, studying notes, and writing questions for myself. It is challenging because this part of the class is different than it was last year, so it is harder to know what to study." (KK, 11/22/99 Journal)
	passages in text(s) difficult to understand	2	7	"Actually, I think the book's really awful. Because we were going over like exponential functions and logarithms and stuff and yet there are really basic equations that they could have put in the book that are not there. So then we had to ask our GSI who doesn't know anything, so it doesn't help much -- the book doesn't explain it" (MT, 10/13/99 Interview)
	representations of mathematical relationships	1	8	"Another thing that they do here a lot that I didn't do in high school, to this point I couldn't…I'm pretty well understanding it now, but everything is graphical. Everything. You have to look at a graph and write a formula for that graph." (DD, 10/23/99 Interview)
T	relationship with teacher	8	17	"I didn't hate math this much until I had him [my GSI]. ... because like last year my math teacher you know tried to help me around test problems and see different ways I could go about it. And he [my GSI] just said well you need tutoring. but that wasn't it because I know it and the GSI said you need tutoring, that's it. I don't have this problem. So he didn't help me at all he made me feel worse." (CA, 10/18/99 Interview)
	basic communication differences	5	15	"The GSI's a little bit hard to understand. Just he has a different way of doing things in terms of how he explains the problems and stuff. ... I don't know it's just tougher adjusting to his teaching style. Especially he has an accent. Heavy accent too so he tends to go off on different angles." (VJ, 11/10/99 Interview)

[17] "Repeat & Emphasis" refers to the number of students who mentioned a difference repeatedly and with emphasis. "Mere mention" indicates how many of the 19 students mentioned this dimension of difference at least once.

(Table 7 continued...)

Category	Difference	Repeat & Emphasis	Mere Mention	Example quote
T	assessment	3	13	"Oh, yeah pretty much irritating for me was my GSI. ... He didn't really know anything but besides that, I don't know, he's give us a quiz and everybody would fail it, we'd get like one right. And he wouldn't give us partial credit on anything, he's give us a 12 point quiz and if it's one thing, the whole things wrong." (MT, 3/26/00 Interview)
	way of conducting activity differs	2	6	"She's from Singapore, but that's not the problem, it's just she, I think she assumes that we know too much sometimes. She'll just, in writing out like her examples on the board she'll put you know two steps down when really she needs to show the step in between them that she's just doing in her head. I think she knows her math she's just not really good at teaching." (BD, 3/28/00 Interview)
	nature of student questions supported by teacher	2	7	"Students would ask her a question and she'd have this look on her face, like you should know this. And she really wasn't teaching the course, like she'll say, you read so you know it, and like we'd ask a question, and she'd look at us like, well you know such and such or she'd make a snide remark and you were just asking questions and say well, I just answered that. Well, can you do an example? Well, there are examples in your book; well, you know we want the GSI to explain it to us so we understand it. So she really didn't teach the class. She'd like put a problem on the board and say do this problem. And we'd do it; well you got it right so you must understand it. Well that might just be a fluke or you know it might be an easy question so we're not understanding what we're doing. That was a problem. She wasn't explaining to us what we were doing and why we were doing it. And then on test we have to do that so it kind of make it, made it difficult to do that." (TM, 3/26/00 Interview)
	constituency of groups	1	15	"I think if we could have picked our groups, the people who you work best with. Because everybody works differently and especially in math there are so many different ways to come up with the solution that if you work with people that you like and even simply that your schedule fit with then it just makes it a lot easier. I think that group homework has potential to help but just the way that it was organized didn't really help for me." (LS, 12/16/99 Interview)
	activities in typical lessons	1	14	"No we really went over a lot of examples and I think that was helpful. I think that's what they don't do a lot in 115 and in 116. Is they don't really go over it, they'll do one example and that'll be the easiest example and then they want you to do, which is helpful cause then you get to do it but then you don't know how to do certain things, so then when they come later you're like I have no idea how to do it. I think if they just did a couple more examples like as if he just did you know like the teacher just did a couple more then it would be more helpful. That's what, in high school it seemed like they did a lot more of those. The teacher showing you so you could actually follow through I thought that was more helpful." (CM, 3/26/00 Interview)

(Table 7 continued...)

Category	Difference	Repeat & Emphasis	Mere Mention	Example quote
SP	final exams do not represent "what we learned"	1	8	"There were some of them [problems on the final exam] that were the same but you, like what you expect on the exam is what things that was put a lot of emphasis on in class. And that's what I expected and that's what I studied the most. And yet I think these problems were more random then the ones that we discussed like in class. Like a lot of things that were emphasized in class some of those didn't even appear on the exam that I studied really hard. Then other things that seemed to be not as important appeared on the exam for more points then I would expect. So I don't know. I don't know whether or not that was my GSI who was misleading us a little bit in what the exams would be like or whether or not I was studying wrong or getting the wrong impression in my class." (LS, 11/4/99 Interview)
C x T	benefits of group homework	9	17	"I had trouble figuring out what was going on so I had to rely on either my classmates or the book, which I did. I also felt the book didn't really clearly explain some of the material. So I think the group study homework sessions helped a lot because then you had 4 guys who were in the same or 3 other guys in the same situation as me. Not really knowing and just going on minimal you know what you can pick up in the book and each person can pick up a different type of concept and together we could make one person who actually knew what they were doing. It helped out." (MM, 3/26/00 Interview)
	presence of group homework	8	18	"The teaching methods aren't that much different. I mean, our class she goes over problems that we have in our homework. Then she teaches the new lesson, and assigns new homework. Um, even the problems in the book aren't that much different at all. But, the group homework is what is I don't think is necessary, period. But that's one big difference." (JC, 9/30/99 Interview)
	disadvantages of group homework	6	14	"My team homework group is the exact antithesis of a team. 'There is no I in team', well in this team there are about 4. We met yesterday and worked on the problems for about an hour then finally decided that it would be a lot easier to just split them up. We decided that if you need help on your problem, go to the math lab." (SB, 2/10/00 Journal)
	typical homework assignments	5	17	"I think that the homework's there [in high school] is easier and the same thing with the quizzes in class because they are over real specific points and I mean it doesn't take much studying for that. I mean if you just pay attention in class and get those, A's in those." (JC, 12/16/99 Interview)

(Table 7 continued...)

Category	Difference	Repeat & Emphasis	Mere Mention	Example quote
C x T	support for developing deeper understanding of mathematical ideas	1	7	"Today I will write about personal work and understanding. We are working on integrals now and it is a little difficult, but I haven't really started to get down to it yet, but when I do, which is tomorrow, it will be cake. In high school we did about a month on integrals. Right now it is a lot more abstract than what we did in high school, but it is getting to the point the long way. In high school we cut right to the chase. I think that in the end this way of learning will be better, but right now it just seems like overkill. I would imagine that once problems start to get harder, the fact that I know all of the details will help me to be able to sort out the methodology in my head." (DD, 12/6/99 Journal)
C x T x SP	required to verbally or in writing explain solutions	10	16	"Yeah, we never had to do that in high school and like 105 is like pretty easy for me cause like I took Calculus last year, I mean some of it's like I have to work at but most of it like I know and that’s like hard for me though, you have to go back and explain something that I feel like I’ve known forever when it’s just…I don’t…I don’t know. It’s just I don’t understand why we have to like explain every single thing like there’s some like quizzes where I’ve got like all the number work is right but I haven’t gotten like a good grade just because I didn’t explain something and that’s really frustrating." (BL, 10/6/99 Interview)

Table 8
What students did NOT notice as different in college math as compared to high school[18]

Category	Difference	Mere Mention	Example quote
C	long-term coherence/connection among chapters or units	9	"Actually the material [in 116] is we pretty much started right where we left off in 115 and then stuff I've had actually before. We, this is still a part of what Calc I was in high school. It's still the pretty much just finding area under the curves and various methods which we were still doing in high school but I think in the next couple week we're starting new stuff." (PJ, 2/10/00 Interview)
	surface characteristics of text	4	"And the book is not all that spectacular on that kind of stuff, so if you didn't understand it, you're just stuck ...You'll miss like key equations or key concepts and they won't highlight them at all. ... Yeah they'll have an example problem and they'll have four paragraphs of explanation and in one line in the center of the paragraph is what they're actually trying to tell you but they don't say that so it takes a lot more deciphering and then a lot of time you still miss it because they word it so strangely you don’t understand it." (MT, 3/26/00 Interview)
	terms in text(s) difficult to understand	4	The most challenging part of these sections is that I have to learn them on my own (through the book) since I cannot understand what my teacher is teaching. Another challenging part of the problems is not being able to understand the vocabulary in the questions and therefore not being able to know what to solve for." (MM, 11/9/99 Journal)
	presence of unrelated or surprising topics or problems	4	"Um…I think we’re starting off what seems to me not even to be Calculus, from what I got out of my senior year…what Calculus was. We’re doing more graphical analysis on what is a function and it is…we are moving at a faster pace I’ve noticed lately because I think it’s in preparation to get to the Calculus. I noticed the section we just did was actually our Chapter 4 in our Pre-Calc, in our book. But I think this course, there’s a lot more emphasis on writing and explaining how you got your answers, the steps you took to get there, and they’re putting more emphasis on that, it’s not just are you getting the right answer." (PJ, 9/30/99 Interview)

[18] Note that this table captures both what students mentioned but without emphasis or repetition AND dimensions that students noticed as not being different from what was experienced in high school.

(Table 8 continued...)

Category	Difference	Mere Mention	Example quote
C	short-term coherence/connection among topics	4	"Well I think as far as some of the teaching methods go, I think the order in which we were taught is different then what we were taught in high school. I remember in high school, say for these integrals. I believe that we were taught to do any derivatives before we were taught to do integrals. And in this class it's kind of like they're throwing the integral at you without teaching any of that, how to really solve it. They're just saying, here's the integral, what this is, is it means that this shows you the area under a curve. And in high school they would teach you how to actually solve an integral and they'd go this will also be helpful for doing story problem's in which you find area under a curve." (PJ, 10/9/99 Interview)
	absence of expected or desired mathematical topics	1	"They are trying to force something that is not going to happen. I mean, they can do better material. I mean, if you are in Calculus, you should be doing Calculus based things, not Algebra II stuff. And I'm just wondering what we're doing in a Calculus class." (JC, 9/30/99 Interview)
	diversity of mathematical topics	0	not mentioned by any participant
T	pace of teaching (by teacher)	11	"So I was kind of confused during class but eh, he was going so fast and I didn't want to like for me I have to take time and just look at the problem for awhile and like see how they got it and how they did it. But he was just like zooming right through it so I was like uh, he's, he asked if anyone had questions but I was like, it's take me longer to, I'd have to look at it myself to understand it." (DJ, 10/19/99 Interview)
	have to do more work on own	10	"Um it's a lot, I think it's a lot different than high school because um, you kind of teach yourself. Well in the beginning it was easy cause it all was review. Now, it's getting harder so it's um, you got to study yourself and you can't always trust the GSI to like teach you because he's not a real great teacher. Because he knows everything and we just really, a lot of us don't know it, so he kind of goes through it kind of quick and um you kind of got to teach yourself." (BE, 10/21/99 Interview)
	duration or importance of activity	6	"I had, like I said before, like 20 minutes to half an hour of lecture in high school and [now] we got a full hour and thirty minutes. ... [The GSI] goes over example problems that are part of the lecture, but its, he'll have us work out problems, but it all goes into the lecture. It's not too bad but sometimes it's hard to pay attention just because that last half hour, that you've been there so long that that door is looking nice." (BD, 10/8/99 Interview)

(Table 8 continued...)

Category	Difference	Mere Mention	Example quote
T	pursuit of multiple solutions to problems	3	"Because, I don't know, my instructor now, he will answer a couple of questions. But he gets kind of an attitude about it. And he tries to explain things, and he always explains the hard way to do it. And he goes, 'Oh, but there are easier shortcuts to do it.' But then he won't show you how to do it. ... And so those of us who already know the shortcuts are showing the other students how to do it because he doesn't bother." (CA, 10/18/99 Interview)
	teacher's use of terms not used in the text	2	"A lot of times he even goes farther then what's in the book, like oh yeah and they got this from some guy 500 years ago and etc. We learn about a whole bunch of different stuff and then it makes us understand the concept we're working on even better." (MT, 3/26/00 Interview)
	sequence of activities	2	"Well I think as far as some of the teaching methods go, I think the order in which we were taught is different then what we were taught in high school. I remember in high school, say for these integrals. I believe that we were taught to do any derivatives before we were taught to do integrals. And in this class it's kind of like they're throwing the integral at you without teaching any of that, how to really solve it. They're just saying here's the integral what this is, is it means that this shows you the area under a curve. And in high school they would teach you how to actually solve an integral and they'd go this will also be helpful for doing story problem's in which you find area under a curve." (PJ, 12/9/99 Interview)
	size of groups for group homework	0	not mentioned by any participant
	nature of questions posed by teacher	0	not mentioned by any participant
SP	pace of presentation of material (dictated by department)	9	"The class is starting to pick up a little bit now. It seems like it is moving faster than 105. It may not actually be moving faster, I think it seems like this because I am less familiar with the material. I can't imagine seeing this stuff for the first time and going over this information as quickly as we do. High school math moved at a snails pace compared to what we are doing now." (SB, 2/24/00 Journal)
	increased anonymity	4	"Yeah. The thing is I wish I had gotten to know more people in the class because where we sat is became our group. ... We only, we sat in, he changed the seating, if I had probably gotten to know more people maybe I could've made more connections and made more friends, gotten more help somehow. I mean I do talk to one person from one of my groups still, we're friends. Well not on a math level but we still keep in contact you know and then I honestly don't know half the people in that class now. Like their names." (DJ, 3/29/00 Interview)
	larger class size	0	not mentioned by any participant

(Table 8 continued...)

Category	Difference	Mere Mention	Example quote
C x T x SP	required to explicitly show all steps in solution	6	"I think there'd be a pretty big difference. The tests in high school were like do this problem, calculate interest of this. And the tests now are how did you get to this, write a paragraph to explain your reasoning. Then like before in high school they said show your work so you can get partial credit. I mean that way if you got it wrong you could maybe get some points. Now you have to show your work to get any credit. I mean a guy in my hall when I was waiting for this last test to come see if the grading was consistent just wrote an answer and they took off half the points for the problem even though it was a right answer, which in high school would not have happened." (KK, 12/13/99 Interview)

Table 9
Summary of flagging based on change in learning approach

	Strategy flag?	Details
CA		
SB		
JC		
BD		
DD		
FD		
GD		
BE	yes	did more work on his own, did homework much more regularly, read book more both before and after class
DJ		
PJ		
VJ		
KK		
BL	yes	did homework everyday before class, read the book, did the practice exams
BM	yes	spent much more time working on math homework, did more work on own, did group work problems in advance
CM		
MM		
TM	yes	studied with peer groups, attended extra help sessions regularly
LS	yes	hired a tutor and met twice a week all year
MT		

Table 10
Summary of flagging based on change in disposition

	Disposition flag?	**Details**
CA	yes	strong **decrease** in enjoyment of math
SB	yes	**increase** in enjoyment of math
JC	yes	**decrease** in enjoyment of math
BD	yes	**increased** appreciation for the difficulty of giving explanations and of teaching math to others and also of how useful these abilities are to the development of his own understanding
DD		
FD	yes	**decrease** in enjoyment of math
GD		
BE		
DJ	yes	strong **decrease** in enjoyment of math
PJ		
VJ		
KK	yes	**increased** enjoyment of math
BL	yes	strong **decrease** in enjoyment of math
BM		
CM		
MM		
TM		
LS		
MT		

Table 11
Summary of flagging from all four analytical lenses

	Achievement flag?	**Differences flag?**	**Disposition flag?**	**Strategy flag?**	**# of yes's**	**Transition?**
CA		yes	yes		2	maybe (no)
SB		yes	yes		2	maybe (no)
JC		yes	yes		2	maybe (no)
BD	yes	yes	yes		3	yes
DD	yes	yes			2	maybe (yes)
FD		yes	yes		2	maybe (no)
GD					0	no
BE		yes		yes	2	maybe (no)
DJ	yes	yes	yes		3	yes
PJ	yes	yes			2	maybe (no)
VJ	yes	yes			2	maybe (no)
KK		yes	yes		2	maybe (yes)
BL	yes	yes	yes	yes	4	yes
BM	yes			yes	2	maybe (no)
CM	yes	yes			2	maybe (no)
MM		yes			1	no
TM	yes	yes		yes	3	yes
LS	yes	yes		yes	3	yes
MT	yes	yes			2	maybe (yes)

Table 12: Transitions summary, Year 1 participants

	Trans?	Transitions Group	Achievement		Differences			Disposition		Strategies	
SB	maybe		N		Y	4	somewhat	Y	increase in enjoyment of math	N	
PJ	maybe		Y	Math ↓↓ , GPA ↓ (2nd term)	Y	6	somewhat	N		N	
VJ	maybe		Y	Math ↑, GPA ↓ (1st term)	Y	2	somewhat	N		N	
BM	maybe		Y	Math ↓↓ , GPA ↓ (2nd term)	N	0	somewhat	N		Y	spent much more time working on math homework, did more work on own, did group work problems in advance
CM	maybe		Y	Math ↓, GPA ↑ (1st term)	Y	7	very	N		N	
TM	maybe		Y	Math ↓, GPA ↓↓ (1st term)	Y	4	somewhat	N		Y	studied with peer groups, attended extra help sessions regularly
GD	N	coast/review	N		N	0	somewhat	N		N	
BE	N	coast/review	N		Y	6	somewhat	N		Y	did more work on his own, did homework much more regularly, read book more both before and after class
MM	N	coast/review	N		Y	4	somewhat	N		N	
CA	N	general	N		Y	1	very	Y	strong decrease in enjoyment of math	N	
JC	N	general	N		Y	4	somewhat	Y	decrease in enjoyment of math	N	
FD	N	general	N		Y	1	somewhat	Y	decrease in enjoyment of math	N	
DJ	Y	negative	Y	Math ↓↓ , GPA ↓ (1st term)	Y	1	somewhat	Y	strong decrease in enjoyment of math	N	
BL	Y	negative	Y	Math ↓, GPA ↓↓ (1st term)	Y	4	very	Y	strong decrease in enjoyment of math	Y	did homework everyday before class, read the book, did the practice exams
MT	Y	negative	Y	Math ↓↓ , GPA ↓ (1st term)	Y	9	somewhat	N		N	
BD	Y	transform	Y	Math ↑, GPA ↓ (1st term); Math ↓↓ , GPA ↓ (2nd term)	Y	2	very	Y	increased appreciation for the difficulty of giving explanations and of teaching math to others and also of how useful these abilities are to the development of his own understanding	N	
DD	Y	transform	Y	Math ↓↓ , GPA ↓ (1st term); Math ↑, GPA unch (2nd term)	Y	9	very	N		N	
KK	Y	transform	N		Y	4	very	Y	increased enjoyment of math	N	
LS	Y	transform	Y	Math unch, GPA ↓ (1st term)	Y	8	very	N		Y	hired a tutor and met twice a week all year