The Navigating Mathematical Transitions Project: Background, Conceptual Frame, and Methodology

Jack Smith
Michigan State University

&

Dawn Berk
University of New Hampshire

for the entire Mathematics Transition Project research team (in alphabetical order):
Dawn Berk, Carol Burdell, Beth Herbel-Eisenmann, Amanda Jansen, Violeta Lazarovici,
Gary Lewis, Courtney Roberts, Jon Star, and Shannon Wellington

Paper prepared for the Navigating Mathematical Transitions Symposium
(Discussant: Jim Fey, University of Maryland)
session #39.11 at the 2001 Annual Meeting of the
American Educational Research Association
Seattle, WA.
April 2001

The research reported in this paper was supported by a grant from the National Science Foundation (REC-9903264). The views expressed here do not necessarily represent the views of the Foundation.

Introduction

This paper was planned and composed to outline the rationale, goals, framing concepts, and methodology of a three-year, NSF-funded project to study the experiences of students who move between “reform-based” and more traditional mathematics curricula and associated pedagogy. It introduces but stops short of presenting the results we have generated in 1.5 years of research at our four school/university sites. Those results and our characterizations of the students who have worked with us are given in a series of site-specific papers also written for this conference symposium (Burdell & Smith, 2001; Jansen & Herbel-Eisenmann, 2001; Lewis, Lazarovici & Smith, 2001; Star, 2001).

The paper is structured into six main sections. In the first, we provide the motivation for studying students’ mathematical transitions in this period of mathematics curriculum reform. Next we present the basic research design that orients our data collection and analysis. Essentially, we examine students’ experiences at four research sites (2 high schools and 2 universities) where students move either from a “reform-based” mathematics program into a more traditional one or the reverse. Third, we describe our methods for learning about our students’ mathematical experiences. Then we present and argue for some of the basic concepts that have framed our data analysis. Fifth, we discuss the methodological issues involved in putting these concepts to work on the data we have collected. Last, we present a few general results that are not specific to particular sites.

Throughout the paper, we use the first-person plural pronoun (“we”) as the subject of many sentences. This choice reflects our intent to describe thinking and decisions that have shaped the work of entire project team. As Principal Investigator the first author made some of these decisions individually and exercised substantial influence in most others. That said, many of the positions recounted here were joint decisions of the entire project team.

Part I: The Need

The publication of *Curriculum and Evaluation Standards for School Mathematics* in 1989 was a watershed event for mathematics education in the United States (NCTM, 1989). More than any other document, the “Standards” shaped a decade of design and discussion in our field (and continues to do so). Chief among its effects has been the design, development, and implementation of elementary, middle, and secondary curricula designed to achieve its image of “mathematical power.” By the late 1990s, these curricula were classroom-ready and were being adopted widely, though not uniformly throughout the country. This was especially the case in the state of Michigan where one middle (the Connected Mathematics Project) and one high school (the Core-Plus Mathematics Project) curriculum were written.

Despite the attention and interest generated by the Standards, the adoption of curricula developed in their image have stirred a great deal of controversy (**Math War citations**). The main root of these controversies lies in how deeply the new curricula have departed from the canonical features of more established (that is, “traditional”) mathematics curricula. The two traditions differ in many ways, but one difference certainly concerns the relative emphasis placed on arithmetic and symbolic computation, the role of standard algorithms, the use of practice (drill) to learn those algorithms. Professional mathematicians and some groups of parents have voiced objections, sometimes very strongly, to the change in emphasis in Standards-based curricula from skill and practice to mathematical meaning and purpose. This change followed from the critique of traditional curricula and classroom practice that is explicitly or implicitly given in the Standards and the new curricula designed around them. Those who feel that past educational practice has “worked” for them question and object when that lessons of past are rejected (or appear to be).

In the debates that have ensued, locally and nationally, advocates for different positions have tended to rely on single cases of student success or failure to advance their more general argument about the “effectiveness” of their respective curricula and practices. One mathematics educator well-seasoned in these debates has aptly dubbed them “killer anecdotes.” They are presented in extreme terms (hence “killer”) and based in expeiences or reports known only to the presenter—so they cannot easily be examined or questioned. “Argument by example” has also been the staple form of journalistic coverage of the debates. Such extreme examples obscure much more than they reveal. Seldom, if ever, are these examples followed by argument or discussion about how they might (or might not) relate to the experiences of the wider student population. When the experience of the wider population is considered, the chief indicator is standardized test scores—by any account a crude indicator of student learning, much their attitudes and experiences more broadly. What is clearly missing from the debate is even-handed information and analysis about how students think about, experience, and evaluate these new curricula. In general terms, the project described here was designed to provide such a body of information and analysis—as fuel for a more grounded discussion and debate about how best to serve the mathematical needs of our nation's students.

It is a truism that we can only think about the experiences that we have actually had. Very few of us can imagine experiences that are deeply different—and this is especially the case for young people. Most of us need contrasts between different sorts of experiences to hold these experiences up to reflection and analysis. In the field of experience we call "school mathematics," any study of students' reactions to "newer" curricula requires first finding students who have also had substantial experiences with more traditional programs. Without that experiential contrast and the reflections likely to flow from it, "math" is just "math." Students' evaluations will generally be limited to what they have experienced.

In fact, the implementation of Standards-based curricula has been "spotty," both nationally and in Michigan where our research was situated. Not only do school districts make their own decisions about which mathematics curricula they will buy and use, their adoptions of curricular materials are often restricted to a single grade band, e.g. "middle school," in mathematics and other subjects. So while a district's middle schools might implement a Standards-based curriculum, their high schools might retain or choose a more traditional program. Students in these districts would then experience quite different curricula within a relatively short time period. In this period, their attention would be drawn to the differences in what these different programs expected them to think, do, and say. So this pattern of "spotty" implementation provides the necessary context for finding and exploring students' experiences, reactions, and evaluations as they move between traditional and Standards-based mathematics curricula. Though it creates difficulties for teachers who may find themselves teaching students who are ill-prepared for the kind of mathematical activity they desire, the spotty implementation of Standards-based curricula has d exactly the right context for studying students' reactions to mathematics curricula that are substantively different.

But what might students actually notice as they move between fundamentally different curricula? Though local discussions and controversies around reform mathematics curricula generate a lot of "heat," they often fail to shed a great deal of "light." Attributions about the nature of both "reform" and "traditional" curricula are often made without clear reference to dimensions of difference that underlie and support the distinction. In preparation for the work described in this paper, we examined three reform curricula to see where we (as mathematics educators) could see salient patterns of difference (Star, Herbel-Eisenmann, Smith, 2000). At the middle school level, we examined the Connected Mathematics Project (CMP) materials; at the high school level, the Core-Plus Mathematics Project (CPMP) materials; and at the college level the Harvard Consortium calculus and pre-calculus materials.[1] Table 1 presents the results of our analysis. Though we have come to see some of these dimensions as more salient to students than others, they constituted our initial position on what might be seen as different by students.

Table 1
Key differences between Standards-based and more traditional
middle school, high school, and college mathematics curricula

Traditional materials	Standards-based materials (CMP, Core-Plus, Harvard)
<u>Fundamental mathematical objects</u>	
Equations & symbolic expressions	Functions and functional relationships represented in tables, graphs, and equations
<u>Typical problems</u>	
"Solve," "factor," "multiply," symbolic expressions; Verbal statements with request to find a numerical value (word problems)	Verbal statements with tables, graphs, and/or symbolic expressions with requests to find values and describe, explain, predict, and interpret.
<u>Typical solution methods</u>	
Complete the correct steps in symbolic procedures in the correct order	Relate verbal statements to tables, graphs, or equations; Compute or manipulate that representation; Interpret the results
The role of <u>practice</u>	
Significant practice on particular problem types	Similarities between problems are less salient; Extended work on fewer, more open problems
<u>Technology</u> for representing and calculating	
Used in balance with pencil & paper computation, which is more highly valued	Supports students' work on most all problems
Typical <u>lesson elements</u>	
Review homework, present new content, provide time for work on next assignment	More variation across lessons; Mix of teacher presentation, small group work, and whole group discussion

Given this framing of the issue of students' reactions to spotty implementation of Standards-based curricula, the question becomes: How do students react and adjust to curricular change in mathematics that shift what they are expected to do, say, and think along the lines outlined in Table 1?

[1] We acknowledge that the Harvard Consortium materials were not directly grounded in the content of the NCTM Standards. In fact, the calculus reform movement (**Douglas**, Tucker, 1990, **other references**) was well underway when the 1989 Standards were published. Yet it remains an interesting fact that the Harvard curriculum for pre-calculus and calculus was structured around many of the same core principles that directed the development of Standards-based K-12 curriculum in the 1990s. This parallel is sensible if we hold in mind that the same theory and research in learning and teaching jointly influenced both movements.

Beyond the stories reported by teachers who taught with these materials and the "killer anecdotes" of media reports and school board presentations, we found virtually no literature on students' reactions to reform—in this or previous periods, e.g., the "new math" period of the 1960s and 1970s. Two noted exceptions are work by graduate students interested in precisely this issue. Candy Baguilat (1998) examined high school students' reaction to an Algebra II course that was structured around contextual problems, multiple representations, multiple solution methods, and no official textbook. She found that students in this classroom reacted quite differently to this "treatment." Some relished the opportunity to express and explore their own thinking; others simply could not understand why the teacher did not tell them how they solve the different kinds of problems they worked on. Rebecca Walker (1999) examined the beliefs of Core-Plus students about mathematics, at the end of their high school program and in the first semester of their college mathematics coursework. One of her university sites used the Harvard Consortium materials in pre-calculus and calculus; one used more traditional calculus and pre-calculus textbooks. She found that the students' beliefs were generally stable across time, though there were two significant changes: Students were more likely to see mathematics as less useful and more centrally involving facts, formulas, and algorithms after a semester of collegiate mathematics. In terms of achievement, no student in her small interview sample (n = 6) struggled with their first semester coursework.

In some prior research the first author and colleagues studied the kinds of understandings of linear relationships developed by 8th grade students who had worked with CMP materials for three years with solid teaching support (Herbel –Eisenmann, Smith, & Star, 1999; Smith, Herbel-Eisenmann, Star, & Jansen, 2000). We found relatively robust understandings of key concepts of slope and y-intercept, but only some elements of these understandings matched the standard textbook conceptualization. For example, students typically used the terms "rise over run" to reason about the slope in graphical context and the "coefficient" or "the number that multiples 'x'" in symbolic contexts. But they also reasoned about slope in tabular representations only in terms of the change in the y-column: "what it goes up/down by." This approach is related to the standard slope formula but not identical to it. They also used non-standard terms to describe graphs that were non-linear (Herbel-Eisenmann, 2000). Given this mix of standard and non-standard knowledge and standard and non-standard terminology, it seemed very likely that much of what these incoming 9th graders knew about linear and non-linear relationships could easily go unrecognized in their high school mathematics classes. In other words, there were likely differences in how students understood and talked about one key content area, and these differences were over and above the potential differences outlined in Table 1.

From this preliminary conceptualization and pilot work, we developed an intuitive sense of our target phenomenon. <u>Mathematical transitions</u> became more likely in the context of fundamental shifts between curricula but were not reducible to them. Indeed, we did not yet know what sorts of differences students would notice and report. We felt a transition required some element of struggle and adjustment, though in what way(s) we were very unsure. With this admittedly vague notion of mathematical transition, we framed three initial research questions in our research proposal to the National Science Foundation:

• What are the characteristics of successful (and unsuccessful) mathematical transitions?

• How do students navigate them? What combination of resources and actions influence success (and failure)? How does the transition process play out over time?

• Based on that analysis, could schools and universities provide additional external resources would support more successful transitions? If so, are these resources feasible and sensible?

As the second question suggested, we did not expect that students' transitions would necessarily become evident immediately after a curricular shift. Rather we planned for experiences that could take much longer to become conscious and be articulated.

This work intentionally foregrounds students' perspectives in the controversy and debate over what we ought to be teaching in school mathematics and how we might achieve those goals. We aim to assemble a fair and balanced set of portraits of students' mathematical experiences as they moved into and out of Standards-based curricula. We did not think that students' experiences should be the only data source or voice in these local, state, and perhaps national discussions. But we do think that students' perspectives should be included, and in their full diversity, for one simple reason: Argument by killer anecdote must be avoided. It fundamentally warps and misconstrues the complexity of students' mathematical experiences, and of teaching and learning mathematics more generally. To achieve our goal, we must represent students' views and experiences as fairly as honestly as possibly, setting aside whatever personal views we may hold as individuals. Further, our portrayal of students' experiences must be accessible to the multiple audiences whose lives are influenced by mathematics curricular reform (parents, teachers, administrators, and policy-makers). Different materials may be required to speak to different audiences, but to make a difference we must speak to those who are unfamiliar with the terminology and patterns of thinking common in mathematics education research.

Part II: The Research Design

Our premise is that passage between a Standards-based curriculum and a more traditional curriculum increases the chances that students will experience a different set of expectations for "good" mathematical thinking, acting, and speaking and (potentially at least) struggle to adjust to these new expectations. If so, such dislocations could be caused by either movement from a Standards-based curriculum into a more traditional curriculum or from a traditional curriculum into a Standards-based program. Either way, students could face a substantial, even dramatic change in expectations.

We decided to look for mathematical transitions at two points in students' mathematical experience: When they enter high school and when they enter college. We wanted at least two educational levels in our design because that would permit comparisons of transitional experiences across age and level of schooling. If, for example, parallels might exist between high school and college students' experience in moving from Standards-based to traditional curricula, our design should permit those parallels to emerge.

Numerous considerations suggested our choice of junior high school to high school and high school to college as our best choice. First, the level of controversy around the reform movement and curricula increases with the students' age and educational level. Fewer complaints arise in elementary school and many more appear by high school, because access to college (and to particular colleges!) is so closely tied to educational opportunity and economic well-being. It seemed appropriate to choose the age levels where the controversy and interest was greatest because those were the contexts where an account of students' experiences might be most useful. Second, older students were more likely to be articulate about the issues and situations we would be asking about. They are also more able to draw on a varied background of school mathematics experiences, thus increasing the chances of interesting contrasts with experiences in their present program. Third, the CMP middle school and the CPMP high school curricula were both developed in Michigan and therefore had been adopted in numerous school districts around the state. Their dissemination provided some potential research sites within driving distance of our project base in East Lansing.

With these considerations in mind, we sought schools and universities that "fit" the following 2 x 2 school site design.

Table 2A:
Basic Research Design
Mathematical Transitions Project

	Type of Curricular Shift	
	Reform to Traditional	Traditional to Reform
Junior high to High school		
High school to College		

The Four Research Sites and Their Mathematics Programs

i. <u>Prescott High</u>, the reform junior high to traditional high school site
The middle school (grades 6–8) where previous work had suggested this project fit the reform to traditional pre-college cell nicely. This school had been a "lead" pilot site in the development of the CMP curriculum, and the teachers were highly knowledgeable and very comfortable with that curriculum. Moreover, it was the only middle school in its district, and it "fed" the only high school (grades 9–12) in that district. Most important, the high school mathematics staff had considered and rejected CPMP and retained a traditional set of courses (Algebra I; Geometry; Advanced Algebra; Functions, Statistics, and Probability; and Pre-Calculus) using a variety of textbooks from Glencoe, Prentice-Hall, and the University of Chicago School Mathematics Program (UCSMP).[2] The high school mathematics faculty was willing to participate in the project because they

[2] In the years just before the project came into the high school, the high school program used UCSMP materials almost exclusively. But that pattern began to change during the first year of the project when new Glencoe (Algebra I) and Prentice-Hall (Geometry) textbooks were purchased.

recognized the differences between their middle school and high school programs, though they were cautious about the project's presumed association with the CMP curriculum and its authors.

The district included a small town and a larger surrounding region of smaller villages and rural, agricultural areas. The population was mostly middle-class, but not affluent. Many heads of households commuted an hour or more to white-collar and blue-collar jobs in larger nearby cities. The student population was almost entirely white. This site provided nearly ideal conditions for examining one sort of mathematical transition. It was also one site (and eventually the only site) where we had detailed understanding of our future participants' classroom experiences from prior work and Beth Herbel-Eisenmann's dissertation work (Herbel-Eisenmann, 2000). Our pseudonym for the high school in this district became Prescott High School.

ii. Logan High, the traditional junior high to reform high school site
As a contrast for Prescott, we searched for and located a nearby high school (i.e., one within a 1.5 hour drive of the MSU campus) that "received" students from a traditional junior high curriculum and taught Core-Plus. Logan High, our pseudonym for this school, paralleled Prescott in a number of ways. The district had only one junior high and one high school so the task of understanding students' junior high school mathematics experiences was easier (though we had much less information than at Prescott). The community, a middle class suburb of a medium-sized city, supported a mix of blue and white collar work but was not affluent. The student population was primarily white, with a smattering of African-American, Hispanic, and Asian students.

Logan had been an early pilot site for the CPMP curriculum, and the mathematics faculty were comfortable with the program and experienced in teaching it. They had from the beginning, however, maintained a two-track mathematics program: CPMP and the University of Chicago School Mathematics Program (UCSMP) materials.[3] The high school employed "block scheduling," so students typically took 4 semester-long courses for 1 hour and 20 minutes each day. The increased class time allowed mathematics teacher to teach about 2/3 of a year's content. This schedule necessitated be "repackaging" the four-year CPMP curriculum into a 5 course sequence (Integrated Math 1–5). Blocking scheduling allowed students to "double up" in mathematics (and other subjects) if they so chose. The high school also offered both AB and BC Calculus. Calculus had typically enrolled students from the UCMSP mathematics program, though this pattern appeared to be changing.

For most students, the choice of high school mathematics programs actually resulted from student choices made at the end of the 7th grade. Students who were considered "advanced" in mathematics at the end of the 6th grade (on average, between 1/4 to 1/3 of a typical cohort) were placed into a traditional Pre-Algebra course. The current textbook

[3] This two-program model was more common than not among the Michigan high schools that had implemented Standards-based curricula. High schools that maintained only one mathematics program that was also reform-based existed but were the exception.

for that course is Glencoe's, though the nature of the course had changed little in recent years. Also, Math 6 and Math 7 (for "regular" students) used traditional textbooks. At the end of their 7th grade Pre-Algebra, the "advanced" students were given a choice of taking the first CPMP course (Core 1) or Algebra I as 8th graders. Most incoming 9th grade students continued with the mathematics program they had chosen. Students who completed Algebra I moved on to Geometry; students who took Core 1 moved into Integrated Math 2. Shifts between mathematics programs were rare.

Thus Logan High had many characteristics we were looking for, though it was not a perfect site. The school taught Core-Plus mathematics to most, though not all students from a "traditional" junior high base. Their implementation and staffing were stable, and they were interested in participating. However, some students were introduced to Core-Plus as 8th graders, thus complicating the task of observing and documenting students' first experiences with CPMP. In addition, we knew much less about the nature of the students' mathematical experiences in their junior high classrooms than we did at Prescott.

We were fortunate to find that the two major research universities in Michigan, University of Michigan (UM) and Michigan State University (MSU), introduce freshmen to calculus and pre-calculus via quite different curricula. This existing difference in approach allowed us to fill out the second row of our design matrix.

iii. <u>The University of Michigan (U-M)</u>, the traditional high school to reform college site U-M offers only three introductory mathematics courses, Pre-Calculus (Math 105), Calculus I (Math 115), and Calculus II (Math 116), all using materials developed by the Harvard Consortium (Hughes-Hallett et al., 1994; Connally et al., 1998).[4] As noted above, the Harvard Calculus materials were largely consistent with, though not explicitly designed on the basis of the 1989 NCTM Standards. The fundamental features of these materials include: (1) extensive use of problems developed around "realistic situations;" (2) greater exposure to and analysis of verbal, tabular, and graphical representations of functions; (3) informal development of key concepts prior the statement of formal definitions and theorems; (4) extensive use of graphing calculators throughout the courses; and (5) extensive group work, in and outside of class. All three courses are taught in the small section format (≤ 30 students), largely by graduate students. Since the number of sections of each course is very large, the Department has developed extensive procedures for supporting these Graduate Student Instructors as they learn to teach. Scores on college entrance tests (ACT and SAT), AP tests, and the Department placement test orient freshmen placement in pre-calculus or calculus, though students may chose "downward" not "upward" in the sequence. Though this reform program has been a stable part of the Department offerings for some years, it is important to note that mathematics majors do not take them. They either come through the honors calculus sequence or place into Calculus III or beyond. All these courses which use both traditional textbooks and teaching methods.

---

[4] Very bright and well-trained high school students can place into Honors sections of Calculus I and II and into Calculus III, which are all taught with more traditional materials and teaching methods. But only a very small number of students do so.

For successful graduates of traditional high school programs, the Harvard Calculus courses at U-M makes quite different demands and causes substantial complaint from students. The Department's guide to students, designed to orient students to the text and the course, explicitly warns students about the likely discontinuities between these courses and their high school experiences (Shure, 1998). Thus, U-M provided nearly ideal conditions for studying mathematical transitions at the college level, cued by a traditional to reform curricular shift. All that was required was to exclude students who had Standards-based experiences in high school.

iv. Michigan State University, the reform high school to traditional college site
In contrast, MSU offers a traditional two semester introduction to calculus that uses the most popular textbook nationwide (Thomas & Finney, 1996). This sequence is required for majors in technical fields (mathematics, natural science, and engineering) but is also taken by students pursuing life sciences, business, and social science majors. Calculus I (Math 132) is taught in small sections (≤ 30 students) by mostly faculty instructors. Pre-Calculus (Math 116) and Calculus II (Math 133) are taught in the large lecture, small recitation section format—though some smaller sections of 133 are also offered. Pre-Calculus (Math 116) uses a textbook that emphasizing graphical and algebraic representations to analyze the properties of different families of functions (Bittinger, Beecher, Ellenbogen, Penna (1997). The Department also offers a two semester alternative introduction to calculus (Math 124 and 126) for students majoring in non-technical fields. These courses use the Harvard Consortium's Applied Calculus textbook (**reference)**, which was developed on the principles summarized above, but with greater focus on problem situations drawn from life science, social science, and business fields. Third, MSU, like U-M, offers an honors calculus sequence for small number of students with excellent training and interest in mathematics. In addition to the pre-calculus and calculus offerings, the Department maintains other lower level courses covering primarily high school mathematics. As at U-M, course placement for freshmen is governed by ACT or SAT scores, AP test scores, and the Department placement test, and students may elect to take a lower-level class, but not a higher-level one.

So MSU provided an appropriate site for locating students who might be coming from reform mathematics programs in high school and into more traditional courses. At this site we would seek students who had used CPMP materials in high school or who were graduates on one nearby high school—that we will refer to as Hogan High—where the mathematics faculty had developed their own reform curriculum and teaching methods "in-house" (see Lewis, Lazarovici, & Smith, 2001 for details). To parallel work at U-M, we would seek only those students with a "reform profile" from high school and who placed into Pre-Calculus, Calculus I (Math 132), or Calculus II (Math 133).

With these four sites as cell entries, our design matrix—summarized below in Table 2B—was complete. As an initial target, we planned to recruit between 20 and 25 students at each site, yielding a total project sample of between 80 and 100 students.

Table 2B:
Basic Research Design: Site Names & Curricula
Mathematical Transitions Project

	Type of Curricular Shift	
	Reform to Traditional	Traditional to Reform
Junior high to High school	Prescott High CMP to Traditional	Logan High Traditional to CPMP
High school to College	Michigan State CPMP & Hogan to Traditional	University of Michigan Traditional to Harvard Calculus

Part III: Data Collection

However "mathematical transitions" might be defined (see Part IV for our conceptualization), we anticipated that we could not restrict our observations and interactions with students to a relatively short period of time (a semester or a year). The effects of transitions could take much longer to emerge and be appreciated by students—that is, "appreciated" enough to report them, in written or oral form. So we planned to "follow" our students for a much longer period of time: Two full years of careful data collection and a half year of reduced study. For college students, we would see them through the often difficult freshman year and the often more settled sophomore year. For high school students, we would follow them through two years of high school mathematics.

Our data collection has had two main components: We have observed teaching and learning in mathematics classrooms, and we have gathered extensive individual data from our participating students about their mathematical experiences. By far, more energy has been spent on the second component. We wanted to gather sufficiently rich and diverse information from each participant that we would be able to determine whether they had (or had not) experienced a mathematical transition—once we defined the term more carefully. We felt that mathematical transitions should not turn on achievement indicators alone (e.g., grades in mathematics classes) or on changes in beliefs or attitudes alone. Both were relevant to students' mathematical experience.

Instead, we gathered data on individual students across six different domains of experience: (1) performance (course grades, test scores), (2) content learning, (3) educational and career goals, (4) daily mathematical experience, (5) beliefs about themselves, learning, and mathematics, and (6) adjustment strategies (should students alter their learning activity). With a cautious distrust of grades as indicators of learning, we included "content learning" as a separate domain of inquiry. We knew that we could characterize so many student' learning in so many different courses, but we did feel that we could target a few key concepts at each level and see how deeply students understood them. The six domains of data collection and the corresponding measures and/or contexts are summarized below in Table 3.

Table 3: Data Collection; Domains and Measures
Mathematical Transitions Project

Domain	Measures
Performance (generally and in mathematics)	Past & present GPA Past & present math grades SAT/ACT scores Quiz and test grades Classroom observation (high school)
Content learning (mathematics)	Problem solving interviews Quiz or test "talk-through" Journal records Classroom observation (high school)
Daily mathematical experience	Journal entries (paper, audio-tape, or e-mail) Classroom observations E-mail communication (college)
Beliefs about mathematics and themselves as learners	Conceptions of Mathematics Inventory (CMI) Patterns of Adaptive Learning Survey (PALS, for high school) Motivated Strategies for Learning Questionnaire (MSLQ; for college)
Education & career goals	Interviews
Adjustment strategies	Interviews Journal records

i. <u>Recruitment of Participants</u>

At each site, we planned to recruit a diverse and representative sample of students: Incoming 9th graders in Fall 1999 at Prescott and Logan and incoming college freshmen at MSU and U-M. Recruitment and selection processes varied somewhat across sites and are explained in greater detail in the companion papers (Burdell & Smith, 2001; Jansen & Herbel-Eisenmann, 2001; Lewis, Lazarovici, & Smith, 2001; Star, 2001). We give a brief overview here. At U-M recruitment was relatively straightforward. It involved a combination of e-mail solicitation and posted flyers and students with the appropriate profile were not hard to find. At MSU, the recruitment task was more difficult because we sought only graduates of certain high schools, principally those we knew had "good" implementations of CPMP.[5] The MSU Admissions Office provided a list of entering freshmen organized by high school and mathematics course placement. E-mail solicitations were sent to all students who appeared to fit our "reform" profile and were enrolled in pre-calculus or calculus (Math 132 or 133). But because many CPMP high schools also taught a parallel traditional mathematics program, we often attracted students who had had "traditional" experiences, and it took time to sort out their individual backgrounds (see Lewis, Lazarovici, & Smith [2001] for details on the problem and its solution). In short, finding the "right" students at MSU was much more challenging.

At Prescott high, some students volunteered at the end of their 8th grade year (May, 1999). Others were recruited in beginning of the Fall semester in their 9th grade mathematics classes. Project staff (Smith and Jansen) described the project, its activities, and goals, and identified interested students. Because we were less familiar with the Core-Plus curriculum and the Logan site, the Fall 1999 semester was spent getting settled with the curriculum and the teachers. Recruitment began in January 2000 and began with presentations in mathematics classes with a high concentration of freshmen. At both high school sites, we sought to develop a sample that was diverse in (1) academic performance, (2) interest in mathematics, (3) gender, (4) social and personality characteristics, e.g., shy vs. outgoing, confident vs. not. At the university sites, it was impossible to exert much control of these characteristics as we had no access to the students prior to their volunteering. In the end, we ended up with moderate diversity at these sites, though we were not able to "choose" it.

At all sites, we met with interested students in small groups to answer their questions and to clarify their duties. With few exceptions, students who expressed interest in the project also became project participants. And for the most part, students who volunteered for one semester have continued to participate. Overall, there has been minimal attrition in our sample—retention being helped no doubt by the financial reward we offered.

ii. Financial Reward

Because we were unsure that anyone—other than our handful of volunteers from Prescott High—would agree to participate, we chose to offer a substantial financial reward for participation: $250 per semester for university students, $125 per semester for high school students. Payments were made at the end of each semester and were reduced if a student did not complete all project assignments (interviews, journal entries, surveys). University students who were not taking mathematics were paid $60/semester for one interview and some e-mail contact. MSU students received a credit to their tuition bill; students at all other sites received checks. One advantage of the reward system was that it provided leverage over students who slacked off on their project work. Generally speaking, most participants were surprisingly faithful about their work and appeared more strongly motivated by the idea that their experience might help other students than by the financial incentive.

[5] CPMP staff were helpful in identifying these schools.

iii. Observations of Teaching

Classroom observations were devoted toward the main goal of characterizing how the curricula were presented to students. It is well-documented "theorem" of educational research that the "enacted curriculum" is rarely, if ever, the same as the "written curriculum." At Logan and U-M we did not want to presume that a written reform curriculum was necessarily taught the way that the designers of the curriculum intended. Likewise, at Prescott and MSU, we did not want to infer that a "traditional" curriculum necessarily led to "traditional" teaching. At the two high schools, we were able to observe each mathematics class where our students were placed many times each semester. This was more difficult to accomplish at the university sites because of the sheer number of sections, the resulting scattering of our participants across them, and difficulties at times of gaining access to these classes. In general, we were able to observe most all classes at least once and many 2-5 times during the semester. At the high school sites especially, these observations also gave us a chance to observe our students working (or not) in their classroom environment.

iv. Interviews

Consistent with our goal of understanding and characterizing students' experiences in their own terms, individual interviews have been the backbone of our data collection efforts. Our interviews serve different purposes. Early in each semester (or year for high school students), interviews focus on their new mathematics course and how they see it in relation to their past courses. At other times, we use interviews to clarify characterizations students have given in previous sessions or in other communication. Toward the end of the semester, we pose problems we have developed to assess students' understanding of key ideas in their coursework or ask students to solve (or resolve) a recent test or quiz problem—also to assess their understanding. For example, one problem assessed how well the university students in first semester calculus could connect the various meanings of the derivative.

v. Surveys of Beliefs

Beliefs were assessed via standard multi-item surveys with Likert scale responses. The Conceptions of Mathematics Inventory (CMI), designed by Douglas Grouws and colleagues at the University of Iowa (Grouws, 1994; Grouws, Howald, & Colangelo, 1996), was used because it was sensitive to the kind of changes that Standards-based curricula might provoke. It is composed of 8 questions on each of seven scales: (1) composition of mathematical knowledge, (2) structure of mathematical knowledge, (3) status of mathematical knowledge, (4) doing mathematics, (5) validating ideas in mathematics, (6) learning mathematics, and (7) usefulness of mathematics. Two surveys developed at the University of Michigan assessed students' motivational beliefs and beliefs about learning. The Motivated Strategies for Learning Questionnaire (MSLQ) was developed for use with college students (Pintrich, Smith, Garcia, & McKeachie, 1993), and the Patterns of Adaptive Learning Survey (PALS), for use with middle school students (Midgely, Anderman, & Hicks, 1995). We used the PALS with our high school students.

vi. Journals

We also wanted to understand how students were experiencing their coursework, day to day and week to week. We wanted to provide students with some means of reporting important events that happened between interviews that influenced their mathematical experience. (This seemed especially important for the college students who we did not usually see in class on a regular basis.) Our method was a paper or audio journal that we asked students to write (or speak) once a week, typically when they were working on mathematics away at home or in their dorm room. We supplied a standard set of questions, situated in their current assignment but we also found that, over time, departures from the standard questions were desirable. At both high schools where we examined the students' journal entries more regularly, we asked them to respond to specific questions that followed up on previous entries. In the best cases, these exchanges approximated a dialogue. In the worst case, we had to nag students to write in their journals and we reduced their semester stipend if they neglected this duty too badly. Overall, we have been pleased with what learned from this source.

Part IV: Fundamental Concepts

A major conceptual challenge for our project was the lack of any clear definition of the target phenomenon, "mathematical transition." Everyone on the project team held and spoke from some intuitive conception, but they were all different, often inarticulate, and not systematically applied. Since no clear definition or meaning was available from our search for prior research, we proposed that one project "deliverable" would be a clear and "operationalized" definition of mathematical transition.

i. Some Existing Notions of Transition

As we have pursued this work and listened carefully to educational discussions, both in schools and in academic research, it has become clear that the term "transition" is a common and variably used term, both in reference to the teaching and learning of mathematics and more generally. So before we present our current working definition of "mathematical transition," we first discuss some existing meanings of "transition" and what we consider their shortcomings relative to our analytic task.

- A transition is the student's ascendance into the next psychological stage. This meaning is tied to a stage-theoretic conception of learning or development. For example, in Piaget's theory of cognitive development, students were seen—at certain points in their development—to be "transitional" between one stage and the next. Some "pre-operational" children would show dissatisfaction with the logic of their responses to "concrete operational" tasks but could not yet apply concrete operational reasoning to solve them. Interestingly, the responses of such transitional children were often weaker than those whose development was less advanced, because confusion and inconsistency resulted from the reorganization of their cognitive structures. This conception is some ill-matched for our purposes because we posit no underlying general theory of mathematical learning or development to ground claims about "transitions." That said, we do analyze changes in how individual students go about learning mathematics, as part of our conceptualization of mathematical transition. Moreover, some students (though a relatively small percentage) report that their current mathematical experiences have led to fundamental changes in their academic goals and methods of learning. Though we will not analyze and report these as stage-like development and transition, these changes in thinking and behavior do appear, at least in some cases, to reflect real developmental change in those students**.**

• A transition is a student's move into the next level of schooling. This perspective targets broader developmental changes in students as they move between different school buildings with new populations of students (usually larger) and new demands for social and academic behavior and skills. For example, Jacqueline Eccles and her colleagues have examined the transitions of students into junior high school as they intersect with the developmental challenges of adolescence (**Eccles references**). This perspective embraces both students (and the full range of their cognitive and affective experience) and the school environments they inhabit (e.g., Eccles' conception of "person-environment fit"). This conception is inadequate for our purposes because there is no inherent connection to the subject-matter of mathematics. We have retained this perspective as a part of our analytic frame (see below) but it is not how we have conceptualized mathematical transitions. In brief, our stance has been to record and analyze aspects of students' experience that do seem tied to their ascendance to the next level of schooling separately from those aspects that more specifically about their mathematical experience.

• A transition is a student's move into some fundamentally new mathematical content. From this perspective, transitions happen to students solely because they enroll in some mathematics course(s). This conception is mathematical in nature but targets only changes in the world, i. e., the mathematics curriculum presented to students. For example, the "transition from arithmetic to algebra" is a common description in mathematics education research, but it generally means only that instruction in arithmetic is more or less over and students now find themselves in an algebra course. Little (if any) description typically is given about how the demands on students' thinking change from their confronting the new content of algebra. For our purposes, what is missing from this meaning is the student's experience (and learning) of that new content. In other words, this view of transition is primarily a change in the world, not a change in the person, and such a conception is inadequate for our purposes. That said, we acknowledge that mathematical transitions are not independent of students' experience of content that is new, different, and intellectually challenging for them. But the primary target of our analysis is their interaction with that content and that challenge.

ii. General Developmental Changes and Mathematical Discontinuity

In developing a more adequate conception of mathematical transitions than these, we have found it useful to characterize two related notions, general developmental changes and mathematical discontinuity. First, we recognize that students' mathematical experiences, whether they encountered traditional ←→ reform curricular shifts or not, are but a small part of students' lives (their felt experience). Students' lives, even those who are concerned about their grades and learning and like mathematics as a school subject, are filled with activities, experiences, and challenges that are (1) non-mathematical and (2) much more important than their experience in and around mathematics classrooms. Though our primary goal was to understand students' mathematical experience, we did not want to confuse our research focus with their primary experiential focus. Generally speaking, high school and college students are preoccupied with all the challenges of understanding themselves in relation to their world and in becoming adults. Learning mathematics is but one small part of that task. On the other hand, mathematical experiences are not separable, experientially or analytically, from the broader range of students' experiences and actions. And this is true whether mathematics is important to the student or not.

Thus we retained from the work of Eccles and others a notion of the developmental challenges presented to students by a shift from one school building, culture, and set of expectations to the next. We began to use the term, "general developmental changes," to refer to changes that students reported about their "new life" in 9^{th} grade or as college freshman that were different from their prior experience as students. These changes were not essentially mathematical in character. Though they might connect to mathematical experience, their origin was not mathematics, and they were more general than students' experience of that subject-matter. For example, high school students frequently reported that they sensed (and welcomed) greater freedom and autonomy accorded to them by their teachers than was true in junior high. The college students also cited the greatly enhanced freedom over how to use their time, whether to study or not, and the increased responsibility that came with that freedom. In both cases, these differences between "now" and "before" were non-mathematical in that they arose more generally from the shape of the overall educational institution, high school/college. Carrying this conceptual box, "general developmental changes," into our analyses allowed us to acknowledge and categorize important aspects of students' experience that intersected with their mathematical experiences but were not specifically mathematical in their character.

The notion of mathematical discontinuity follows from out premise that the introduction of Standards-based curricula might make mathematical transitions more likely. If the differences that we saw and summarized in Table 1 were real, then students should report some of them as they described how they saw "math last year" compared to "math this year." But which would they report, and would they report differences that we had not listed? We reasoned that if a student experienced a mathematical transition, then part of that experience might involve the conscious notice of differences in enacted curricula (present vs. past). If students noticed and could articulate significant differences of this nature, in our view they experienced a mathematical discontinuity. They noticed that the mathematical world they were asked to participate in had changed. It remained for us to operationalize this term (see below for details).

iii. <u>A Working Definition of Mathematical Transition</u>

With the conceptions of general developmental issues and mathematical discontinuity in hand, we could turn to the task of specifying more carefully what a mathematical transition was and how we would decide if any project participant had experienced one. Developing this working definition took a great deal of time, discussion, and reconsideration. We eventually settled on a model that identified 4 constituent factors: (1) academic performance in mathematics, (2) disposition toward mathematics as a subject, (3) approach to learning mathematics, and (4) mathematical discontinuity. In general, <u>academic performance</u> is achievement in mathematics coursework (grades). <u>Disposition</u>

towards mathematics is students' affective orientation to the subject and is expressed by three indicators: (i) their intentions to take further mathematics coursework; (ii) their attitudes toward the subject, and (iii) their beliefs about mathematics and about learning mathematics. Approach to learning means the autonomous actions that they undertake, both physical and mental, to learn mathematics. The qualifier, "autonomous," requires that the actor has some freedom in executing those actions; it excludes change that only involves actions mandated by teachers. Our working definition became:

> Students have undergone a mathematical transition if our analysis indicates significant change on 2 or more factors of their mathematical experience.

An important feature of this definition is its flexibility. The criterion of "any two" makes it possible that students would "qualify" as having experienced a mathematical transition for different reasons. This choice reflects our view that all four factors were important dimensions of students' mathematical experience but no one of them was sufficient. For example, if a student reported significant differences between the past and the present but (1) continued to achieve at the same level, (2) maintained the same disposition toward the subject, and (3) did not change his/her learning approach, we did not think this should count as a case of mathematical transition. In our view, the student noticed a difference but little else changed. Likewise, if a student's performance in mathematics dropped but their disposition and learning approach did not change, and they did not report significant differences, we were not comfortable calling that a mathematical transition. He/she might have simply found the mathematics more difficult to learn—without seeing any difference in how the mathematics was presented in the textbook or taught.

Clearly, the key term in this definition (and indeed, in each factor) is "significant." While it made sense not to count any change, no matter how trivial, as significant, it was not trivial to decide what would count. We had to address this issue for each factor in our model.

- Significant change in performance. We wanted to avoid two pitfalls in defining change in mathematics performance: (1) failing to take into account how students were doing in their other subjects, and (2) considering only one semester's performance after the curricular shift. After much discussion we settled on a formula that compared the difference between consecutive mathematics grades (e.g., 12th grade vs. college semester 1) to the difference between corresponding overall grade point average (e.g., 12th grade vs. college semester 1). If the difference of the differences was greater in absolute value than .5, we decided that the student's mathematics performance had changed significantly—relative to his/her overall academic performance—in that semester. Symbolically, we can describe the formula for any given semester as follows:

$$|\Delta \text{ Math Grade} - \Delta \text{ GPA}| \ ?\geq? \ .5$$

Since we have 3 semesters of performance data in the "new" mathematics program, we decided that a second-order difference greater than .5 in any semester (i.e., "before" vs. "semester 1," "semester 1 vs. semester 2," and "semester 2 vs. semester 3") counted as "significant change in mathematics performance" for that student. As we have just begun to apply this standard to our data, we are not fully convinced it is optimal and in time may revise it. But this is standard that we have applied in all our site-specific AERA analyses.

• Significant change in disposition. We have defined a significant change in the disposition toward mathematics to be a non-trivial change in either their attitudes or their beliefs. Data on attitudes comes primarily from interviews; data on beliefs comes from survey data. Changes in attitude or belief may be accompanied by changes in plans to take more mathematics, but such changes are neither necessary nor sufficient. We elected to exclude changes in intended mathematics coursework because we felt that many factors could influence students' plans and changes to their plans, and some might not involve their mathematical experience. Currently, criteria for "non-trivial" change in attitudes and beliefs are not clearly and objectively defined. We currently require that a convincing argument must be made to the whole project team for any student to be scored as a significant disposition change. In the absence of convincing data, we score the disposition factor as "no change."

• Significant change in approach to learning. We define change in the student's approach to learning mathematics in terms of that student's observed or reported strategies to learn mathematics. A significant change on this factor is indicated by evidence that the student has (i) tried out new learning strategies in response to current coursework, (ii) started using his/her existing strategies in new and non-trivial ways, or (iii) stopped using some "tried and true" strategies. The primary data for significant change in learning approach comes from the interviews.

• Significant differences. Because one important objective of this research is to understand how students view curricula that look quite different to mathematics educators, we directly ask students where they see differences between their current math course and their previous mathematics program. Our questions are posed in general, (e.g., "what is different…?") and more specific terms (e.g., "how does a typical day in math class this semester/year differ from last year?"). With these direct question we cannot count any difference response as significant. Instead, we have defined the differences reported by a student as significant if any one of them is (i) reported spontaneously, OR (ii) repeatedly mentioned, OR (iii) given particular emphasis or attributed particular impact by the student (in more than one interview session). The main principle here was to require some indication of importance for and/or impact on the student. Consistent with our judgment that some aspects of curricular reform might be seen by both high school and college students, we developed a uniform coding scheme for differences (see below and Appendix 1).

Part V: Analysis Plan

In this section we present other methodological tools we have developed to pursue the analysis of mathematical transitions. These include a more detailed set of research questions, an outline of our units of analysis, our standard for characterizing the teaching we have observed, and our scheme for coding the differences that students report between their current and their prior mathematics program.

i. Revised Research Questions

As framed in the proposal, our original research questions were three: (1) What are the characteristics of successful (and unsuccessful) mathematical transitions? (2) How do students navigate them? What combination of resources and actions influence success (and failure)? How does the transition process play out over time? (3) Based on that analysis, could schools and universities provide additional external resources would support more successful transitions? If so, are these resources feasible and sensible?

We ultimately expect to address these questions, but to do so we have found it necessary to “unpack” a number of additional intermediate questions. Framing the following questions has helped us bridge the gap between our data and original questions.

(1*) What is current classroom instruction like? That is, what is the intended (text materials) and enacted (teaching practice) mathematics curriculum for participating students?

(2*) What do students notice as different and how important are these differences? In the other words, who notices a mathematical discontinuity, and what is the character of that discontinuity?

(3*) What are students learning in their mathematics classes?

(4*) Which students experience mathematical transitions and which do not?

(5*) For those students who do experience a mathematical transition, what strategies and resources do they try out to adjust to the discontinuity? What strategies and resources do they ignore? What are the consequences of trying out those strategies and resources?

Research question 1* was necessary because we could not assume that the written curriculum would translate easily into the enacted curriculum. In particular, we did not want to assume that implementation of a Standards-based curriculum necessarily meant either that (a) the curriculum was taught as the designers intended, or (b) the teaching was aligned with current reform perspectives on mathematics teaching (i. e., the principles outlined in the NCTM’s Professional Teaching Standards (NCTM, 1991). In the next section we present the main conceptual tool we used to assess our students’ current instruction in mathematics. Research question 2* was necessary for two reasons. We were interested in what sort of differences students would notice and report as an important dimension of our analysis. But the difference analysis was also needed because it was one factor in our concept of mathematical. Research question 3* reflected our desire not to reduce mathematics learning to performance in courses (grades) but to assess students’ mastery of at least some key concepts. Research questions 4* and 5* were essentially more careful restatements of the content of research question 2.

ii. Units of Analysis

It should be clear from the presentation above that all three main conceptual terms, general developmental changes, mathematical discontinuity, and mathematical transition, are framed with the individual student as the basic unit of analysis. That is, individuals experience general developmental changes (or not), mathematical discontinuities (or not), and mathematical transitions (or not). Thus much of our work involves careful analysis of the data that we have gathered on individual students. However, this analytic focus at the individual level does not preclude other kinds of analyses. We believe that the aggregate experience of each sample (each cell in the 2 x 2 design matrix) has meaning. Both high school samples are significant proportions of their respective school cohorts. Therefore we expect to draw inferences about the general impact of specific curricular shifts on students at those schools. By contrast, our university samples represent much smaller proportions of their respective cohorts. In addition, the MSU site sample is not necessarily representative of its respective cohort because we targeted entering students with a particular curricular profile from high school—either Core-Plus or Hogan High's "in-house" curriculum. We will not claim that their experience has been representative of their site cohort as a whole, most of whose members had more traditional high school mathematics experiences. We will argue that their experience is representative of other MSU students whose high school experiences were also structured the same or similar "Standards-based" curricula and teaching.

We also expect to compare aggregate results across sites, exploiting the design features of the 2 x 2 matrix. For example, comparisons across high schools (Prescott vs. Logan) and across universities (U-M vs. MSU) may help to identify effects that are more due to age and level of schooling than to curricular or teaching issues. In a complementary way, comparing the two reform-traditional sites (and likewise, the two traditional-reform sites) may allow us to find other effects that are, at least in part, due to the kind of curricular shift and are somewhat to entirely independent of schooling level.

In sum, we intend serious and detailed analysis of individual students' experience. But we also expect to make inferences and judgments about the experience of the relevant population at each site from the analysis of our sample. Finally, we plan to compare effects within the 2 x 2 site matrix, both within age/schooling level and type of curricular shift.

iii. <u>Assessing the Enacted Curriculum</u>

In order to address research question 1* above, we needed both a sufficient number of observations of lessons taught to project students and some means of describing and comparing these observations. At the Prescott High, all teachers who were teaching project students have been observed about twice a week for the entire school year. At Logan High, we observed teachers twice a week for Spring 2000, once or twice a week in Fall 2000, and about once a week in Spring 2001. The decreasing number of observations were due to the greater scattering of students across classes.[6] At MSU and U-M, observations were much more difficult to carry out with similar frequencies. First, it was challenging to gain access to the sections where our students were placed. In both sites, human subjects considerations and professional courtesy required that we contact each instructor (faculty or graduate student) and ask permission to observe. This process of contact and project description took time. In some cases at both sites, we failed either to gain access or to complete the observations due to shortage of project staff. When we did gain access (this was true in the vast majority of cases), we completed one observation at U-M and two to five observations at MSU. Though such limited observations preclude

[6] We were fortunate at Prescott that the school administrators helped us group project students in a smaller number of classes than they would have naturally placed into.

our characterizing some teachers in any detail, even single classroom observations have proven useful. They have allowed us, for example, to evaluate what our students tell us about a typical day in their current mathematics class.

At all sites, project staff acting as observers took field notes that focused on what the teacher said and did in class and how students were asked to participate. To make sense of these field notes, we developed a crude teaching model to use as a “baseline” for our descriptions and comparisons. That is to say, the model was a rough description of teaching that we could use to measure each of our project teachers’ practice in terms of their deviations from the model. It does not reflect our individual or collective sense of what is optimal or even productive classroom practice. It is only an explicit and common metric for comparing the observed practice of many different teachers. The model lists explicit teacher actions that we could directly observe and expectations for students’ actions that we could infer from our observations of teachers and students. The model, summarized below in Table 4, was identifies actions and expectations for a typical day of instruction.

Table 4: “Baseline” Teaching Model
Mathematical Transitions Project

Teacher Actions	• Undertake the following activities in the order listed. • Check and evaluate students' homework. • Solicit students' questions from homework and produce solutions (on the board or overhead). • Display the new content for the day and provide example problems and solutions. • On homework and “new” problems, explain how to execute solution procedures. • Evaluate the validity of students’ own methods whenever they are presented. • Provide time for students to solve similar problems. • Describe in advance the content of quizzes and tests in significant detail.
Teacher’s Expectations of Student Actions	• Listen to teachers' presentations of problem solutions. • Refrain from talking to peers, except possibly during problem solving at the end of the lesson. • Take notes on the content presented by teachers. • Answer "small" (i.e., "what's the next step?") questions from the teacher. • Pose “how-to” questions to the teacher about his/her solutions as needed, e.g., “how did you do that step?” and “how did you get that number?” • Solve problems in class that use the procedure or method presented that day. • Study the notes and commit them to memory for tests and quizzes.

This model was developed out of what we were seeing in our classroom observations of teaching; it was not an a priori model. There was clear diversity in what we observed, within and across sites, but there were also some very typical elements. Within the distribution of typical teaching practices we were observing, we wanted a model to express something like the "conceptual mean" of that distribution. That our "baseline" model looks so traditional is an early indication of a finding we summarize briefly below: Across all sites, including Logan High and U-M, we found many elements of traditional instruction.

Some specific comments about elements that appear and elements that do not appear are warranted. We elected not to include group work and use of graphing calculators, not because those elements were uncommon (they were frequently observed), but because there was such dramatic variety in how these elements of practice were deployed. We could not find an easy way to describe typical practice with respect to either element. Also, student questions, beyond the procedurally oriented "how-to?" variety, are missing, and beyond that, elements describing richer student participation in discourse are absent. Deeper, more open-ended, and conceptually oriented questions from teachers are likewise absent, as are teachers' consideration of alternative solutions to problems. All of these discourse-related elements were excluded because they were not commonly observed. Finally, we considered extending the model to list students' expectations of teachers (in part because we were learning a lot about that from our participants). But that move seemed difficult and a step away from the practice of teaching, so we did not take it.

vi. Categorizing Differences

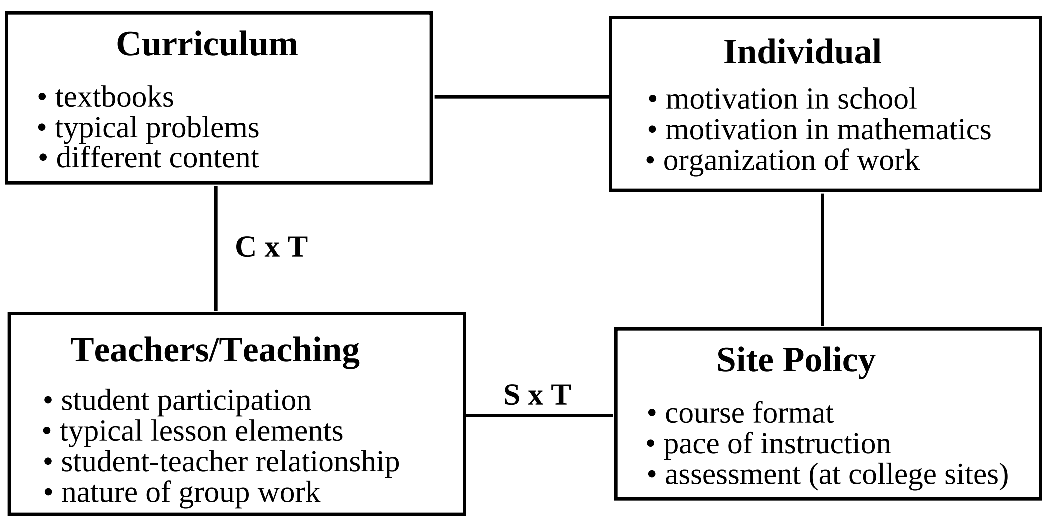
Research question 3* targets the differences that students notice and report between their previous experience of school mathematics and their current experience after the curricular shift. The differences that our participants began to report were, as expected, quite various and sometimes pushed on the boundaries of what we considered "mathematical." More or less spontaneously, we began to consider the origin of these differences: What we might justifiably infer was their source. Initially, we developed three main source domains of these differences: (1) changes in the written curriculum, (2) changes in teachers and/or teaching practice, and (3) changes due to site policy.

The first two domains match the familiar educational distinction between curriculum and teaching. "Site policy" emerged primarily from our work with the college students, who reported strong effects of organizational decisions that were made and sustained by the Mathematics Department as a whole, not by individual instructors. So, for example, an issue of "site policy" was the departmental decision to mandate that all instructors in all sections of a specific course "cover" a specific and fixed portion of the assigned text. Because such decisions were not made by individual teachers but had strong effects on students, they seemed to merit a separate domain in our differences coding scheme.

As our analysis continued, we began to see the need for a fourth domain: Differences that originated in changes in individual students themselves. That is, some students reported that they experienced their mathematics work differently because of changes in their motivation, how they thought about school, mathematics, and their lives, or how they related to their teachers and peers. Such changes in individual students could have only a weak relationship to their mathematical experience, yet we decided to add this domain to our scheme because it seemed no less the source of perceived differences than the other categories above.

As with any analytic scheme developed to "code" and represent complex phenomena involving many individuals, not all the reported differences were easily and simply placed in one of these 4 domains. Some appeared to rest more sensibly at the intersection (or interaction) of two or more of them. For example, we consider differences in assessment (when teachers design their own) and differences in typical homework assignments as originating in the interaction of Curriculum x Teaching. With assessment, we reasoned that teachers chose test and quiz problems from collections provided by the curriculum (or modeled their own problems on similar problems in the curriculum). With homework, we reasoned similarly that the curriculum provided choices of what to assign but teachers made those choices. Thus Figure 1 below the 4 main domains in our differences analysis scheme and what have been thus far the two-factor interaction domains. The actual difference categories—that is, our name for each difference mentioned by students—sorted by domain are listed in Appendix 1. This listing also includes a small number of "residual" difference categories.

Figure 1: Analytic Scheme for Origins of Differences
Mathematical Transitions Project



vii. Constructing Cases

Because the individual is our fundamental unit of analysis and because we are collecting such a rich corpus of data from each participating student, we are presently compiling and processing our diverse data into individual case studies of each and every participating student. We intend these case studies to (1) richly capture students' experience in their own terms and (2) support further analysis within and across sites as outlined above in "Units of Analysis." When complete, the cases can be accessible as narratives, permitting all members of the research team to read and digest data from sites where they were primary data collectors. Eventually, the cases can also be edited down into examples that can be presented to the various audiences we have targeted.

The cases are constructed around a temporal framework that encompasses all the participants' experiences in the project, their experiences at their prior level of schooling (junior high or high school), and further back in some instances. We have designed a case frame for each site with appropriate slots for summarizing different dimensions of students' experience. These frames are somewhat site-specific but also share many characteristics, especially the two high school frames and the two colleges frames. The case frame for the U-M site is attached in Appendix 2.

Part VI: Some General Empirical Patterns

The goal of this paper has been to set up, but stop short of presenting our emerging results based on 1.5 years of data collection. Those results are reported, by site, in the four companion papers from this Symposium (Burdell & Smith, 2001; Jansen & Herbel-Eisenmann, 2001; Lewis, Lazarovici, & Smith, 2001; Star, 2001). But some findings are so general and/or methodological in nature that we summarize them briefly here.

• <u>Diversity of experience is the rule</u>. Should anyone think that all students experience curricular reform in mathematics in the same, or even similar ways, they should think again. We expected diversity in students' experiences but have been surprised by its range and depth. This diversity plainly exists within site, even the high school sites where students have experienced the same mathematical environment and culture. Perhaps because our participants have had extensive experiences in school mathematics (8 years and 11-12 years respectively) and have formed their own views of what works for them, they have reacted to the shift into and out of reform curricula in very different ways. Put somewhat differently, they have reacted (or not) to different aspects of the enacted reform (or traditional) curriculum. Our task as analysts is to search for the elements of commonality in this sea of individual differences. And fortunately, we have found some patterns of commonality within this sea of individual differences.

• <u>Students' ability to be articulate about different dimensions of their mathematical experience is limited</u>. Put simply, students are much less obsessed with analyzing the details of their mathematical experience than we are as researchers. They express their reactions to their mathematical environment in more global terms, and only the most salient (and painful) aspects of their experience are expressed immediately and clearly in interviews and journals. Therefore, one main data collection task has been to frame questions that allow students to describe their experience in greater depth and detail than they do we pose our initial very general questions, e.g., "How does your experience in your math class compare with last semester's?" Thus our response to students' limited articulation has been to become better interviewers.

The crucial step in this process to learn how different aspects of students' experience get conflated in their responses and to pose follow-up questions that help them to clarify more precisely the reality they are trying to express. Similarly, when students offer explanations of their experience (that is, when identify factors of influence) we pose questions to test whether or not other factors of influence are involved. Overall, our data are still heavily dependent on participants' self-report, but we systematically try to test and refine their self-reported experience. In most cases, we have been quite successful to encouraging into the open more careful, thoughtful, and detailed data than was initially produced. That said, no one should think that getting all (or even most) students to talk about their mathematical experience in a clear and comprehensible way is a trivial task.

• <u>The past is not over; its meaning is reconstructed in the present</u>. For us and for most of our participants, multiple occasions to describe and interpret experience has been essential. Our data collection plan was designed with an eye toward the possible gradual pace of mathematical transitions. We wanted to provide time for students' reactions and assessments of curricular shifts to emerge. But an additional benefit of 2.5 years of data collection (with 1.5 years expended so far) has been the multiple opportunities to hear students revisit their past experiences, in some cases to substantially adjust and enrich their interpretations of that experience, and for us to be able to pose more clarifying questions. One issue is <u>completeness</u>. It has simply not been possible to learn, in one or two interviews, about the full complexity, depth, and nuance of students' experiences, past and present. We have benefited greatly from the opportunity to return to ambiguous and not-yet-addressed issues after reviewing the content of previous interviews. Another issue is the <u>revisions</u> that have seen in students' accounts. Their reactions and evaluations at time t_1 sometimes do not match their parallel accounts at time t_2, because intervening experiences have changed their thinking. Though this means that we will never have the final story from and about our participants, it does mean that our cases represent "considered" accounts that are the product of many occasions for students to express and describe their mathematical experience.

References

Baguilat, C. A. (1998). **practicum paper title.**

Bittinger, M. L., Beecher, J. A., Ellenbogen, D., & Penna, J. A. (1997). Pre-Calculus: Graphs and models. Reading, MA: Addison-Wesley.

Burdell, C. & Smith, J. P. (2001). **2001 AERA paper, Logan High analysis**.

Connally, E., Hughes-Hallett, D. H., Gleason, A. M., et al. (1998). Functions modeling change: A preparation for calculus (preliminary ed.). New York: John Wiley & Sons.

Grouws, D. A. (1994). Conceptions of Mathematics Inventory. Iowa City, IA: University of Iowa.

Grouws, D A., Howald, C. L., & Colangelo, N. (1996, April). Students conceptions of mathematics: A comparison of mathematically talented students and typical high school algebra students. Paper presented at the Annual Meeting of the American Educational Research Association, New York.

Herbel-Eisenmann, B. (2000). How discourse structures norms: A tale of two middle school mathematics classrooms. Unpublished doctoral dissertation. Michigan State University.

Herbel-Eisenmann, B., Smith, J. P., & Star, J. (1999, April). Middle school students' algebra learning: Understanding linear relationships in context. Paper presented at the 1999 Annual Meeting of the American Educational Research Association Meeting, San Francisco, CA.

Hirsch, C., Coxford, A., Fey, J., & Schoen, H. (1996). Core-Plus Mathematics Project. Everyday Learning Publishers.

Hughes-Hallett, D. H., Gleason, A. M., et al. (1994). Calculus. New York: John Wiley & Sons.

Jansen, A. & Herbel-Eisenmann, B. (2001). **2001 AERA Paper, Prescott High analysis**

Lappan, G., Fey, J., Friel, S., Fitzgerald, W., & Phillips, E. (1995). The Connected Mathematics Project. Palo Alto, CA: Dale Seymour Publications.

Lewis, G., Lazarovici, V., & Smith, J. P. (2001). **2001 AERA Paper, MSU analysis**

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

1991 NCTM Teaching Standards

2000 PSSM

Shure, P. (1998). Student’s guide, Math 115, Calculus. Ann Arbor, MI: University of Michigan, Department of Mathematics.

Smith, J. P., Herbel-Eisenmann, B.,, Star, J. & Jansen, A. (2000, April) Quantitative pathways to understanding algebra: Possibilities, transitions, and disconnects. Paper presented at the 2000 Research Pre-Session of the Annual Meeting of the National Council of Teachers of Mathematics. **Location**.

Star, J. (2001, April). “Reform” at the collegiate level: Examining students' experiences in reform calculus. Paper presented at the 2001 Annual Meeting of the American Educational Research Association Meeting. Seattle, WA.

Star, J., Herbel-Eisenmann, B., & Smith, J.P. (2000). Algebraic concepts: What’s really new in new curricula? Mathematics Teaching in the Middle School. 5, 446–451.

Thomas, G. B. & Finney, R. L. (1996). Calculus and analytic geometry, part I (9th ed.). Reading, MA: Addison-Wesley.

Tucker, T. W. (Ed.) (1990). Priming the calculus pump: Innovations and resources. Washington, DC: Mathematics Association of America.

Walker, R. (1999). Students conceptions of mathematics and the transition from a Standards-Based reform curriculum to college mathematics. Unpublished doctoral dissertation. University of Western Michigan.

Math Wars citations

Appendix 1
Differences Coding Scheme

Curricular Differences

i. <u>Surface characteristics</u>
• appearance and/or packaging (e.g., color; layout; number and size of volumes, etc.)

ii. <u>Mathematical characteristics</u>
• comprehension of textbook's terminology (easier, harder, etc.)
• comprehension of passages in the text (descriptions, explanations, etc.)
• typical problems (e.g., more "story" than "number" problems or vice-versa)
    • ease/difficulty of comprehension (understanding what the problem asks for)
    • understanding of content required (e.g., more or less, deeper vs. shallower)
    • perceived value or relevance to student
• direction provided for solving problems (e.g., explicit solution procedures for specific problem types (highlighted boxes) vs. general verbal descriptions vs. no guidance)
• representations of mathematical relationships, other than verbal (e.g., tables of values, graphs, equations, diagrams)
• content is different (at the "top level," e.g., geometry as opposed to algebra)
• content is presented in greater depth and detail (in contrast to "new" content)
• content is more diverse mathematically
• some topics or problems seem non-mathematical (that is, outside of the boundary of "math" as defined by the student)
• absence of expected topics (e.g., algebraic manipulation)
• short-term coherence/connection among topics (within chapter or unit)
• longer-term coherence/connection among chapters or units (over a semester or year)

Teachers/Teaching Differences

• teacher-student relationship (e.g., accessibility, trust, care, contact outside of class)
• basic communication problems (literal comprehension: e.g., student cannot hear, understand, and follow the teacher's speech)
• classroom management (e.g., difficulties in focusing, following, or participating because of different classroom norms)
• teacher's typical lesson
    • activities in typical lesson (elements include: checking homework, lecture, experiments or data collection, individual problem solving, small group problem solving, large group discussion)
    • sequence of activities
    • duration or importance of activities
    • organization, direction, or shaping of activity (e.g., nature of class discussions)
• teacher's expectations for student participation (what teachers expect students to say and do)
    • nature of teacher questions
    • nature of student questions invited by teacher
    • discussion of alternative solutions to problems
• nature of group work

- frequency of use (continuum from “never” to “a lot”)
- size of groups (pairs, trios, fours, or larger)
- selection (how groups are set up, e.g., student- or teacher-chosen)
- perceived value (both “negative” and “positive” value)

• instructional pace (when set by individual teacher, not the Department as a whole)
• teacher’s fidelity to the curriculum (how closely the teacher “follows” the book)

Differences due to Site Policies

• instructional pace, when set by Department decision
• class size

Note: if class size difference is mentioned, list also the effect(s) felt by the student.

• duration of class meetings
• duration of courses (e.g., semester vs. year-long)
• assessment (when the content of the assessment is determined by the Department coordinator or committee, not the individual teacher)

- kind of assessment (in-class tests, take-home tests, group projects, performance assessments, etc.)
- problems on assessment (e.g., like or unlike what was “covered” in class)
- time to complete assessment
- assistance provided during assessment
- overall difficulty (as seen by the student)
- “additional” assessments, e.g., “gateway” tests)

Differences due to Individual Changes/Development

• motivation to learn/achieve in school generally (presuming the student does not attribute motivation change to others, e.g., teachers)

Note: In both motivation categories, list source of motivation change if known, e.g., change in student’s goals.

• motivation to learn/achieve in mathematics specifically (presuming the student does not attribute motivation change to others, e.g., teachers)
• ability to organize time and effort
• resilience to academic challenges and difficulties

Curriculum x Teachers/Teaching

• nature of expected solutions to typical problems

- amount of verbal explanation
- detail in “showing work” (numerical and symbolic steps to the solution)

• assessment (when individual teachers have substantial discretion to design their own)

- kind of assessment (in-class tests, take-home tests, group projects, performance assessments, etc.)
- problems on assessment (e.g., like or unlike what was “covered” in class)
- time to complete assessment
- assistance provided during assessment
- overall difficulty (as seen by the student)

• typical homework assignments

- number of problems
- difficulty of problems

Site Policy x Teachers/Teaching

- changes in what students are expected to do on their own

Other

- composition of the class (who is there in the class with the student)

Note: we are scoring changes in how graphing calculators are used in class (e.g., nature of use, frequency of use) but we have not placed this category into our scheme.

Appendix 2:
Case framework for U-M

I. High School Experience

Hometown, High School, and Courses

A paragraph or two on where the student is from, what high school they attended, and which courses were taken in grades 9-12. Include where courses are AP or not. Include any comments that the student made about their hometown, high school, math track, and the AP class or track. Include any comments that students made about the curriculum in use at their school.

A paragraph about students' grades and test scores, including math grades, SAT/ACT scores, and/or AP test scores. Include this table (which Jon will fill in later):

High school GPA:
12th grade math grade (fall):
12th grade math grade (spring):
SAT/ACT score:

Instruction

A paragraph on the student's best and worst teachers, including characteristics of these teachers that made them good or bad. Include any other comments students made about high school math teaching.

Class structure

A paragraph on what a typical math class in high school looked like. Include any comments on group work, homework, technology use, tests, exams, or students' enjoyment, effort, or participation in math class.

II. U-M Math Experience- First semester

Math course and instruction

A short line about the student's fall term math course at U-M, including course number and name (105 = PreCalculus, 115 = Calculus I, 116 = Calculus II) but not GSI's name, section number, or course meeting times.

A section on students' perception of this class as a whole. Include any comments made about the GSI, how the course was taught, or anything else related to instruction. Also include any comments about the student's effort in this class and attitude toward the class.

Homework

A section with anything that student said about homework, including both individual and group homework. Include comments about how often student did individual homework, what group homework sessions were like (for the various groups that student was in during the semester). Include any comments that student made about having to give explanations on homework.

Assessment

A section with anything that the student said about quizzes, tests, exams, and gateways. Include this table (which Jon will fill in later). Include any comments that student made about studying for exams or quizzes.

Group homework average: Quiz average: Midterm #1: Midterm #2: Final exam: Final letter grade:

Our observation(s) of student’s class

A section on our observation(s) of this class. Include the date of our observation. Include any comments that appeared in the original case study about instruction, the GSI, student participation, class format, and assessment. If no observation was made, write "We were unable to observe JS's class during this semester."

Our observation of student’s homework group

A section on our observation(s) of student's homework group. If no observation was made, write "We were unable to observe [student name] homework group during this semester."

Career goals

A paragraph on what student said in the fall interviews about his/her major and career goals.

Differences

A section of what students noticed about differences between this U-M math class and his/her high school math class. Begin this section with students' answer(s) to the question of how different U-M math was (somewhat different, not at all different, very different) -- most students were asked this question twice in the fall term. Then identify the major areas of difference that the student commented on. Also, indicate whether these differences seemed major or minor for the student.

Keys to success

What did the student identify as the "keys to being successful" in this course? (Students were asked this during the 3rd interview of the fall semester. If student did not have a 3rd interview, write "Student was not asked about this issue."

III. U-M Math Experience- Second semester

Math course and instruction

A short line about the student's winter term math course at U-M, including course number and name (115 = Calculus I, 116 = Calculus II, 215 = Introduction to Differential Equations) but not GSI's name, section number, or course meeting times.

If the student did not take math in the winter semester, write "JS did not enroll in math for the second semester." and write a paragraph explaining why the student said he/she made this decision. If this is the case, skip the rest of this section.

A section on students' perception of this class as a whole. Include any comments made about the GSI, how the course was taught, or anything else related to instruction. Also include any comments about the student's effort in this class and attitude toward the class.

Homework

A section with anything that student said about homework, including both individual and group homework. Include comments about how often student did individual homework, what group homework sessions were like (for the various groups that student was in during the semester). Include any comments that student made about having to give explanations on homework.

Assessment

A section with anything that the student said about quizzes, tests, exams, and gateways. Include this table (which Jon will fill in later). Include any comments that student made about studying for exams or quizzes.

Group homework average:
Quiz average:
Midterm #1:
Midterm #2:
Final exam:
Final letter grade:

Our observation(s) of student's class

A section on our observation(s) of this class. Include the date of our observation(s). Include any comments that appeared in the original case study about instruction, the GSI, student participation, class format, and assessment. If no observation was made, write "We were unable to observe [student name] class during this semester."

Our observation of student's homework group

A section on our observation(s) of student's homework group. If no observation was made, write "We were unable to observe [student name] homework group during this semester."

Career goals

A paragraph on what student said in the fall interviews about his/her major and career goals. Comment on any differences between students' responses to this question in the winter semester as compared to the fall.

Differences

A section of what students noticed about differences between this U-M math class, the fall term U-M class, and his/her high school math class. Identify the major areas of difference that the student commented on. Also, indicate whether these differences seemed major or minor for the student. Identify whether the student thought that the winter semester class was more different or more similar to the fall class (and why).

Different approach

A paragraph on anything that the student said about how he/she approached this class differently, particularly based on his/her fall term experience.

IV. Transitions

Discontinuity and Transition

A section on transition. Were the differences noticed by the student in the fall and winter semesters sufficient to say that the student experienced a discontinuity? Identity the major "bullet" points about the students' transition. Identify how the student adapted to these changes and whether or not these adaptations were successful.

Achievement summary

You can leave this section blank. Include this table (which Jon will fill in later):

Change in GPA (Fall)
Characterization
Change in math grade (Fall)
Characterization

CMI changes

Include this table (which staff will fill in later):

Comp
Structure
Status
Doing
Valid
Learning
Useful

MSLQ changes

Include this table (which staff will fill in later):

Intrinsic
Extrinsic
Task Value
Control
Self-Efficacy
Test Anxiety
Rehearsal
Elaboration
Organization
Critical Thinking
SRL
Time/Environment
Effort
Peer Learning
Help Seeking

V. Summary