Studying Mathematical Transitions:
How Do Students Navigate Fundamental Changes in Curriculum and Pedagogy?

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Introduction

In the decade since the 1989 publication of NCTM's *Curriculum and Evaluation Standards*, curricula consonant that vision for pre-college mathematics education have been written and implemented in many U. S. elementary, middle, and high schools. Evaluation and assessment studies are beginning to focus on how these curricula perform relative to traditional curricula in supporting student learning and positive attitudes (e. g., Hoover, Zawojewski, & Ridgway, 1997; Schoen, Hirsch, & Ziebarth, 1998). But the implementation of these curricula has been often "spotty," nationally and regionally. Many school districts have chosen Standards-based curricular materials for one or more level of their K–12 system while retaining older curricular materials that reflect less of the Standards vision at other levels. Often, such non-systemic implementations reflect substantial differences, within communities and between school buildings, on how mathematics is best taught and learned.

These "spotty" implementations can create conditions where students experience very different expectations for what it means to think, know, and do mathematics. One curriculum may value and reward students' ability to explain their thinking, work productively with other students, undertake large-scale inquiry relatively independently, while the previous (or subsequent) program of curriculum and teaching does not. Yet very little attention has been paid to the studying the effects of these potential dislocations for students. How do students see such fundamental changes in expectation? How is their learning and attitudes toward mathematics affected? How do they adjust to changes when they recognize them? These are the sort of questions we have designed the Mathematical Transitions Project (MTP) to address.[1] This 3 year project is examining students' transitions at four sites (2 high schools and 2 universities) where students move between programs with "traditional" expectations for mathematical work and those with expectations more consonant with the NCTM Standards.

The Context: The Mathematical Content and Curricula

Though dislocations may occur for students at any point in their K–12 (or K–16) experience, they are particularly likely in two content domains: Algebra and calculus.

[1] The official name of the project is "Navigating Mathematical Transitions: Students' Adjustments to Fundamental Changes in Curriculum and Pedagogy."

Prior research on learning has indicated that learning how to use the algebraic symbolism with meaning and effect and learning to think about the rate of change and accumulation of functions have been difficult for students using pre-Standards curricula (Thompson, 19xx, other cites). Equally important, algebra and calculus have been two important sites for Standards-based curriculum reform.

Algebra

A central objective of middle and high school mathematics is to introduce and develop the study of algebra. Except for one year of geometry, there is a familiar algebraic path to calculus, e.g., Pre-Algebra, Algebra I, Algebra II, Pre-Calculus. Until recently, algebra has been a set of processes for solving equations and manipulating expressions. Problems typically were calls to “factor,” “simplify,” “multiply,” “reduce,” and “solve.” Variables were unknown numbers and functions were expressions in X on the other side of “Y =.” Graphing equations in two variables followed extensive work with one variable equations. Application problems (“word problems’) were very difficult; many students struggle to understand the quantitative relationships they described.

But Standards-based middle and high school curricula introduce algebra as processes of modeling change in realistic situations. Functional relationships where one quantity co-varies with another (and different families of these relationships) become the central objects of study. They are analyzed using tables of values and graphs as well as algebraic expressions and equations. Linear relationships (constant rate) provides the foundation for learning about inverse, exponential, and quadratic relationships. Small group work and graphing calculators are integral components of mathematical work. The Connected Mathematics Project for middle school (Lappan, et al., 1995) and the Core-Plus Mathematics Program for high school (Hirsch, Coxford, Fey, & Schoen, 1996) are two examples of this approach to algebra. Both are widely implemented in the state of Michigan.

The deep and substantive differences between these two approaches to teaching and learning algebra increase the likelihood that students will face dislocations if they move between them. Students schooled in functional relationships in contexts will have to adjust to a greater focus on decontextualized symbols and to new ways of understanding and talking about equations. Students schooled in numerical and symbolic equation-solving will have to broaden their focus and adjust to thinking about functional relationships in context. Either way, the expectations and competencies that students bring forward may be poorly matched to new classroom expectations.

Calculus

An analogous situation exists with calculus. The central objective in calculus is to develop tools to analyze the behavior of functions, specifically, their rates of change and accumulation. Until recently, calculus textbooks emphasized symbolic representations of functions, e.g., “$3x^2 + \sin(x)$.” The goal was to learn how to carry out the symbolic operations of differentiation and integration on them. Much of this work involved learning specific procedures for specific classes of functions. But frustration with student performance and learning and the development of symbolic manipulation tools led to a wave of reform in calculus content and pedagogy (Tucker, 1990). One curriculum developed as part of this reform movement was the Harvard Consortium’s Calculus and

Pre-calculus materials (Hughes-Hallett, Gleason, et al., 1994). The principal object of study was still function, but much more emphasis was given to representing functions in tables and graphs, to problems typically containing more verbal content; and to the use of graphing calculators and small group work.

As we did with algebra, we argue that this calculus reform (which reflects so many dimensions of the algebra reform sketched above) increases the likelihood of dislocations for students. Those who move from equations-based high school programs into the Harvard calculus (or pre-calculus) will likely find very different problems, methods, and expectations for "doing math." Likewise, students coming from a functional relationships-based program into a symbolic calculus course will find the focus narrowed substantially to equations, expressions, and symbolic procedures. In order to be successful, students may find that serious adjustments in their mathematical thinking and practice are necessary.

Based on our "reading" of these curricula (from CMP, Core-Plus, and Harvard Consortium), we distilled out some major dimensions of difference between them and more "traditional" approaches to the same content. These are presented in Table 1 below. Later in the paper, we use this framework to structure our discussion of differences that students notice and report.

Table 1
Differences between "Standards-based" and "Traditional" Curricula

"Traditional" approaches	"Standards-based" approaches
Fundamental mathematical objects	
Equations & symbolic expressions	Functions and functional relationships represented in tables, graphs, and equations
Typical problems	
"Solve," "factor," "multiply," symbolic expressions; Verbal statements with request to find a numerical value (word problems)	Verbal statements with tables, graphs, and/or symbolic expressions with requests to find values and describe, explain, predict, and interpret.
Typical solution methods	
Complete the correct steps in symbolic procedures in the correct order	Relate verbal statements to tables, graphs, or equations; Compute or manipulate that representation; Interpret the results
The role of practice	
Significant practice on particular problem types	Similarities between problems are less salient; Extended work on fewer, more open problems
Technology for representing and calculating	
Used in balance with pencil & paper computation, which is more highly valued	Supports students' work on most all problems
Typical lesson elements	
Review homework, present new content, provide time for work on next assignment	More variation across lessons; Mix of teacher presentation, small group work, and whole group discussion

The Conceptualization: The Notion of Mathematical Transition

We have used the term “dislocation,” somewhat ambiguously above, to refer both to differences in expectations for thinking and doing mathematics and to their effects on students. But for clarity, we introduce separate terms these processes. We use the term, <u>mathematical discontinuities</u>, to refer to marked differences between students’ prior expectations for thinking and acting mathematically and how they find they are expected to think and act in their current classroom. Such discontinuities may occur because of new mathematical content, a new teacher, or movement into a new school building, but they refer to students’ <u>experience</u> of these changes—not to the changes themselves, considered apart from the students. <u>Mathematical transitions</u> are students’ conscious and unconscious responses to those discontinuities: How they respond (or not) to them, and how they understand the results of these responses.

Moreover, we distinguish those processes that intersect with mathematics as a domain of study from more general issues of transition and development that do not. For example, rising 9^{th} graders can experience important changes as they move out of middle school and into high school. They may (and do) experience both an initial sense of exhilaration with their new freedoms but also a sense of dismay at their new obligations and responsibilities. Rising college students likewise face the same issues, in different forms and sometimes much more intensely. Many struggle to find a workable balance between exploring who they are in social terms and accomplishing their academic work. We have found these distinctions helpful in sorting through the various issues that emerge in our data.

<u>Research Questions</u>

Initially, we framed our research around the following three questions:

- What are the characteristics of successful (and unsuccessful) mathematical transitions?
- How do students navigate them? What combination of resources and actions influence success (and failure)? How does the transition process play out over time?
- Based on that analysis, could schools and universities provide additional external resources would support more successful transitions? If so, are these resources feasible and sensible?

But we quickly found that in order to position ourselves to address these questions, we needed to "unpack" them a bit and consider some subsidiary questions, in order to connect questions to incoming data. These include:

- What is current classroom instruction like? Or, somewhat more technically, what is the intended (text materials) and enacted (teaching practice) mathematics curriculum for participating students? How does it compare to their prior instruction?
- What do students notice as different in their current mathematical experience and how important are these differences to them? In our conceptual terms, who notices a mathematical discontinuity, in either written curriculum or teaching practice?
- What are students learning in their mathematics classes?
- Which students experience mathematical transitions and which do not?
- For those students who do, what strategies and resources do they try out to adjust to the discontinuity? What strategies and resources do they ignore? What are the consequences of trying out those strategies and resources?

In particular the "learning" research question reflects our view that it would be problematic to consider issues of "success" and adjustment without addressing whether students' mathematical understanding are growing in important ways.

The Inquiry Plan: Design & Methods

Our overall plan of inquiry has been to locate 4 sites (two high schools and two universities) where mathematical discontinuities and transitions are likely, based on their curricular implementations; recruit a sample of participants; and "follow" those students' experiences for 2+ years of work in mathematics. At two sites (one high school and one university), students were moving from "traditional" curricula to "Standards-based" curricula. At two sites, this directionality was reversed. We hope this 2 x 2 design (see Table 2 below) will help us understand and sort out issues that are site-specific from those that more generally concern mathematical discontinuities and transitions ushered in by curricular change.

Table 2
Sample Design

	Mathematical Discontinuity Type	
Location in K—16 Schooling	Reform to traditional Junior high to high school	Traditional to reform Junior high to high school
	Reform to traditional High school to college	Traditional to reform High school to college

To spell out our research approach a bit we first provide an overview and then considerable detail on our 4 research sites. Then we present our activities with our participants and the kinds of data we have gathered from those activities.

Our Four Research Sites & Initial Student Samples

Table 3 presents a very telegraphic overview of the curricula at our 4 research sites, the classes we observe, and our current participant samples.

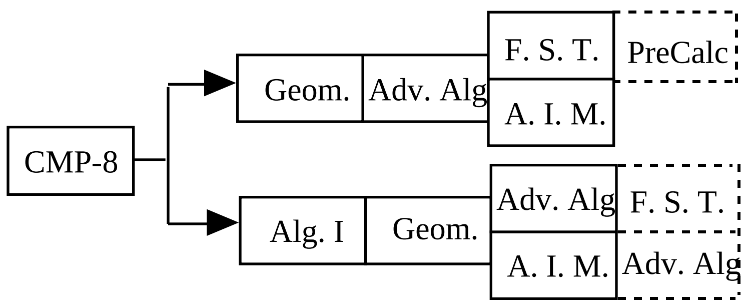
Table 3
Sample Overview

Site A High School [year long courses; 50 minutes/day] CMP → UCSMP, Glencoe, Prentice-Hall Algebra I & Geometry classes Current sample size: 23	**Site B High School** [semester courses; 70 minutes/day] Glencoe, Merrill, Prentice-Hall → Core-Plus Core I & Core II classes Current sample size: 20 & growing
Michigan State University [semester courses; 3 or 4 hours/week] Core-Plus → Pre-Calculus, Calculus I [Bittinger, et al.; Thomas & Finney] Current sample size: 19 (Core-Plus: 10)	**University of Michigan** [semester courses; 4 hours/week] various traditional curricula → Pre-Calculus, Calculus [Harvard Consortium] Current sample size: 21

<u>Site A</u> is a small school district that serves a small town and the surrounding rural area. The district population is largely white and middle class but not affluent. Site A's single

middle school served as a pilot development site for the Connected Mathematics Project (CMP) materials. Their mathematics teaching staff remains knowledgeable of and satisfied with that curriculum. Middle school mathematics is not tracked, and all students take 3 full years of CMP.[2] In prior work (MAI citation), one of us (BE) conducted extensive classroom observations in 8th grade classrooms with a particular focus on teachers' and students' use of mathematical language. These provide the empirical basis for our grasp of (1) our participants' previous mathematical experiences, and (2) differences between their prior and mathematics teaching. Students rising into the high school make their own decisions about which 9th grade mathematics course to take, with input from their family and their 8th grade mathematics teacher.

Three years of high school mathematics are required for graduation. As the Figure below illustrates, rising 9th graders who start with Geometry typically move on to a second algebra course ("Advanced Algebra"), continue with "Functions, Statistics, and Trigonometry," and finish with "Pre-Calculus," if they choose a 4th year of mathematics. An alternative terminal course, "Advanced Intermediate Mathematics" (AIM) was designed by the high school staff to help prepare students for Michigan's high school mathematics test. Students who start in Algebra I have similar choices, though they would have to double-up to reach Pre-Calculus. In the first quarter, Geometry students review algebra topics with textbook pages from an older Prentice-Hall text.



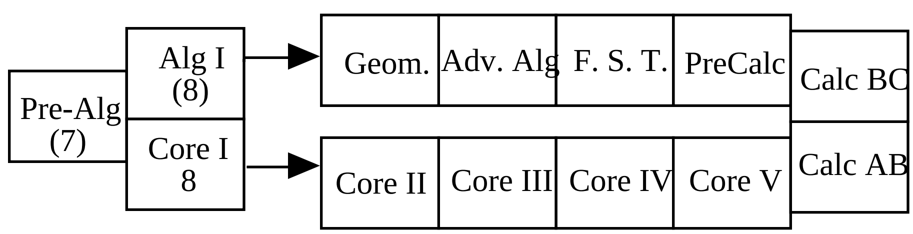
Algebra I students began the year with the University of Chicago School Mathematics Program (UCSMP) text but switched to new textbooks from Glencoe in November. Geometry students were given a new text from Prentice-Hall after they finished their algebra review. The graphing calculators used at the middle school are generally not available to students at the high school. There have also been changes in staffing. Two of 5 high school mathematics teachers were new hires for Fall 1999. One was entirely new to teaching; the other had 12 years experience teaching mathematics, mostly in private schools. Three teachers, including both new staff members, teach 9th graders; we observe all three.

From an initial target of 25, we work with twenty-three 9th grade students (12 boys, 11 girls) at Site A. About half volunteered at the end of their 8th grade year. Eleven are taking Algebra I; 12 Geometry. One is a special education student who receives special assistance for his difficulties with language. Of these 23, only one elected not to re-enroll in the project in Spring 2000.

[2] Beginning in the 1994/95 school year, the district's elementary schools began implementing TERC's Investigations in Number, Space, and Data, starting in the 5th grade and working downard.

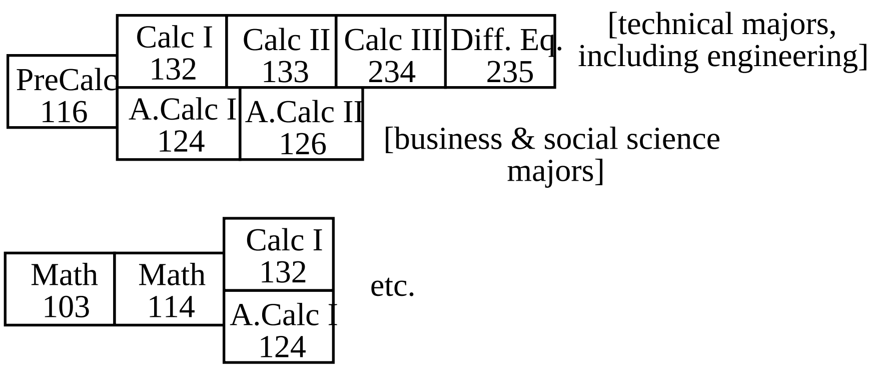
Site B is mixed urban and suburban district representing part of a medium-sized Michigan city. The student population is predominantly white and middle class with some African-American, Hispanic, and Asian students (typically 1 to 3 students per minority group per class of 25 students). The single junior high school teaches General Math, Pre-Algebra, and Algebra I, in grades 6 through 8. The current textbooks are supplied by Glencoe, though previous texts have included Merrill's Pre-Algebra. The higher performing 1/3 of 7th graders take Pre-Algebra and are given the choice of Algebra I or the first Core-Plus course (Core I) in the 8th grade. In 1998/99 there were two sections of 8th grade Core I. We have no direct observations of our participants' 8th grade teaching at the junior high; Site B was not "found" quickly enough to do this. We do plan to conduct some observations of 7th and 8th grade teachers on the assumption that their teaching has not changed significantly in a year.

The one high school was an early pilot development site for Core-Plus. Yet, as the Figure below indicates, there are two more or less separate mathematics programs at the high school. Graduates of Algebra I in 8th grade take the more familiar course sequence using UCSMP textbooks, which leads to Calculus (AB or BC) in 12th grade if they choose to double-up in mathematics. Graduates of Core I continue with that sequence of courses, with the same Calculus options as 12th graders. The remaining 2/3 of graduating 8th graders begin the Core sequence as 9th graders. Graphing calculators are integral to the Core courses, beginning with Core I. Most of the staff of 6 mathematics teachers teach both Core and traditional courses. The high school employs a block schedule (four 1:20 periods each day), and students change courses at mid-year. As a result, doubling-up in a subject area is common and creates few scheduling problems. We began our classroom observations in Core I in Fall 1999 and expanded these in Spring 2000 to include both Core I and II classes, since some 9th graders took Core I as 8th graders. We do not observe in the traditional courses.



We are still in the process of filling out our 9th grade sample at Site B with students who are taking their mathematics in the Spring semester. Nearly all of our volunteers are taking Core I or Core II and did not take mathematics in the Fall semester. We do have 3 students who doubled-up as freshman, two taking Core III, one Core II.

Site C is Michigan State University, which teaches its large enrollment Calculus course with the most common and venerable textbook (Thomas & Finney, 1996), in small sections (Math 132, Calculus I) and larger lectures (Math 133, Calculus II) formats. As at most US universities, MSU also offers a host of entry-level courses "below" Calculus that review high school mathematics. In between, Pre-Calculus (Math 116) is taught in a combined large lecture and small discussion section format, using a text organized by families of functions defined by equations (Bittinger, Beecher, Ellenbogen, & Penna, 1997). The content of Math 116 is also taught in a two-semester course, Math 103/114, at a slower pace. Use of graphing calculators is promoted in Pre-Calculus but limited in Calculus. The Department also offers a two semester “Applied Calculus” sequence (Math 124/126) for business and social science majors using the Harvard Consortium textbook (**cite**). These are terminal courses. Students wishing to major in technical fields typically take the 132/133 sequence. Engineering majors must take at least two additional courses beyond 132/133, typically Math 234 & 235. Freshman access to mathematics courses is controlled by the Department Placement test (essentially an algebra test),[3] students’ ACT Math scores, and (if applicable) their AP Calculus test scores. Students can (and do) elect, however, to take courses “below” the course specified by the Department.



Actually, the figure above represents only four of the most common pathways through mathematics coursework at MSU. Given the proliferation of 100-level courses below Math 116, we focused our observations and recruiting in three courses—Math 116, 132, and 133. In Fall 1999 we recruited 18 freshman who were currently enrolled in either 116 or 132 who also graduated from a high school that used Core-Plus materials. We had no volunteers from Math 133. In Spring we added one additional volunteer from Math 114. Of these 19 participants, 9 were women; 10 were men. We found, however, that many Michigan high schools (like Site B) that use Core-Plus materials also offer a traditional mathematics course sequence. Of our 19 volunteers, only 10 used Core-Plus materials in the majority of their high school courses. To address this limitation, we will recruit a second cohort (more carefully) in Fall 2000. From our Fall sample, all but 3 “re-enlisted” in Spring 2000, and all 3 non-returners were from non-Core backgrounds.

<u>Site D</u> is the University of Michigan, which teaches the Harvard Consortium’s Calculus and Pre-Calculus materials in all freshman mathematics courses.[4] Calculus (Math 115/116) is a year sequence; Pre-Calculus (Math 105) is a semester course. Both are taught in many, many small sections of 25-30 students. In contrast to a more traditional approach, the Harvard materials (Hughes-Hallett et al., 1994; Connally et al., 1998) present every topic geometrically, numerically, algebraically, and verbally (“The Rule of

[3] There are minimum required scores for entrance to each 100-level course.
[4] With the exception of freshmen who place out of both Calculus I and II.

Four") and with formal definitions and procedures developed from work on practical problems ("The Way of Archimedes"). Group work is required in and out of class.[5] Staffing 100+ small sections of Math 105, 115, and 116 strains the Department's capacity to train the graduate students who teach the vast bulk of these classes. Some graduate students come into the Department with no teaching experience; many come from other countries, languages, and cultures. Students who complete Math 116 and take more mathematics move on the usual course options, but these are not taught on the principles of the Harvard Consortium. The content is represented algebraically; the primary method of instruction is large class lecture plus recitation section; and group work is no longer assigned.

PreCalc 105	Calc I 115	Calc II 116	Calc III 215	Diff. Eq. 216

In Fall 1999, we recruited 19 freshmen, 5 from Pre-Calculus, 10 from Calculus I, and 4 from Calculus II. Ten were women; 9 were men. One student had taken 1.5 years of Core-Plus mathematics and switched to the more traditional mathematics sequence at her high school. We also observed the teaching in their classrooms at least once during the semester.[6] Of these 19 participants, all but two "re-enlisted" in the project for the Spring semester. Eleven of these 17 also enrolled in mathematics again.

Assessment Domains and Data

Our efforts to document and understand students' experiences with changing mathematical expectations must avoid two obvious pitfalls. On the one hand, we need to avoid the prospect of examining only the surface of students' experiences, either in cognitive (e.g., simple analyses of course performance) or affective terms (e.g., how students feel about their work). On the other, we clearly cannot locate and capture every single potentially important event, in classrooms and outside of them. Hard choices are necessary, but the emerging portrait of students must be sufficiently rich and detailed that we do not exclude, from the start, important factors of influence.

In our attempts to find this balance, we have chosen to assess our participating students on a relatively broad range of issues. These include: (1) their achievement (the scores they earn on quizzes, tests, exams, and standardized mathematics measures like the ACT, (2) their learning of specific mathematical content that are central to their courses, (3) their everyday experience of their mathematics work, (4) their beliefs about mathematics and about themselves as learners, (5) their career and educational goals; and (6) their strategies for adjusting to changing expectations (once those expectations are identified). Mathematics achievement is a relatively easy domain to manage. It is mostly a matter of collecting the appropriate information from teachers and students. Content learning is more difficult for it turns on our view of what is central and our selection (or design) of appropriate problems. Thus far, we asked students to solve (or resolve) textbook problems or to tackle problems of our design, both in individual interviews. We

[5] With the exception of one section where the instructor elected not to assign group work.
[6] The primary limitation here has been one of visiting so many different sections of the 3 courses, though two instructors declined our request to observe.

use paper journals and e-mail reports to document students' day to day experience. Here the challenge is getting some participants to accept the responsibility and the discipline of the task. We assess students beliefs with two standard instruments—the "Conceptions of Mathematics Inventory" (CMI) designed by Douglas Grouws and colleagues at the University of Iowa (Grouws, 1994; Grouws, Howald, & Colangelo, 1996) and either the Patterns of Adaptive Learning Survey (PALS) (Midgely, Anderman, & Hicks, 1995) for the high school students or the Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich, Smith, Garcia, & McKeachie, 1993) for the college students. These latter two surveys assess students' motivation and learning strategies. Finally, students' career and educational goals and the strategies they employ to adjust to changing expectations are identified in individual interviews.

Table 4
Domains and Measures

Domain	Measures
Achievement	High school GPA SAT/ACT score, overall & math Course quizzes and test grades
Content learning	Quiz or test "talk-through" Problem solving in interviews Journal records
Daily experiences	Journal records (paper, tape, or e-mail) E-mail messages
Beliefs about mathematics and self as learner	Conceptions of Mathematics Inventory (CMI) Patterns of Adaptive Learning Survey (PALS; Sites A & B) Motivated Strategies for Learning Questionnaire (MSLQ, Sites C & D)
Personal goals, education & career	Interviews
Adjustment strategies	Interviews Journal records

Some Preliminary Results

Though our Year 1 data collection is on-going, we report below our preliminary results in two areas: changes in mathematics achievement and significant differences in mathematical experience that students report.

Achievement Changes: The Fall 1999 Math Report Card

Table 1, given below, summarizes what we know about how our participants' mathematics grade changed from the 1998/99 academic year to Fall 1999 semester at 3 of our 4 sites. We define a "significant increase" in achievement to be at least one full number (or letter) grade increase, e.g., a "C" to a "B" or a "3.0" to a "4.0." (A "significant decrease" is defined symmetrically.) "No change" means that students' received exactly the same grade on both occasions. "Marginal increase" (or decrease) indicates an increase (or decrease) of less than a one full number (or letter) grade. We are still in the midst of collecting achievement data from our MSU participants, so our results there are incomplete. Most students at Site B did not take mathematics in Fall 1999, so we do not yet have achievement (or achievement change) data for these students.

Table 5
Change in Mathematics Achievement by Site

Site	Significant Increase	Marginal Increase	No Change	Marginal Decrease	Significant Decrease
Site A HS*	0	6	5	6	4
MSU**	0	1	2	1	4
U-M	1	2	4	5	6

* Data is missing for 2 students, both taking Algebra I.
** Data is reported above for only 8 of the 19 MSU participants, though 7 were Core graduates.

As Table 5 shows, 17 of the 23 students at Site A experienced little change in their mathematics performance from 8th grade. Ten of these were taking Geometry; the remaining 7 were taking Algebra I. Most performed well in mathematics before and after the move to high school. Only 3 of these 17 earned a "C" grade in 9th grade;[7] none received a lower grade. No one at Site A raised their mathematics performance by a letter grade or more. Three of the four whose grade decreased significantly were Algebra I students. Two of those were approaching a failing grade. Two other students, whose data is missing from Table 1, were also doing very poorly in Algebra I (though this may or may not represent a change from 8th grade). In addition, data from our Algebra I students' 3rd quarter performance suggests that those who were beginning to struggle by mid-year are doing even more poorly midway through the Spring semester. Thus, while there were not huge downturns in either course, more Algebra I students were struggling than Geometry students. We note here, as below, that drops in achievement could result from many factors, including students' motivation and engagement.

At U-M the results were similar, though more students' grades fell significantly while 1 student's grade was significantly. Generally speaking, our participants found college academic work harder. Seventeen of 19 students (89%) experienced a drop in their overall GPA from high school to the first semester of college. Moreover, changes in overall GPA correlated highly with changes in mathematics achievement. The 4 students taking Math 116 (Calculus II) did well before and after the college transition, both in overall GPA and mathematics. But more than half of the Math 115 students (5 of 9) experienced a significant drop in their mathematics grade.[8] Four of these 5 also had a significant drop in their overall GPA. Of the 5 Pre-Calculus students, only one student received a significantly lower mathematics grade; the performance of the others was close to their high school grades. One interpretation of these results is that (1) Calculus II students were talented enough to adjust to the change in expectations, (2) Calculus I

[7] That is, a "C+," "C," or "C–."
[8] One of the original 10 students enrolled in Math 115 dropped the course at mid-semester.

students were less so and more tied to high school patterns of work and thinking, and (3) Pre-Calculus students did not suffer because they already knew the material from high school.

Across all three sites, we see what we might have expected—a drop in achievement as students move further along in their mathematics education. Yet, we also see exceptions to that pattern. Some students struggle immediately (a suggestion confirmed in our interviews) and some do not (at least immediately); some raise their performance. What Table 5 does not show, that our other data do, is that one semester is too short a time frame in which to assess mathematical discontinuities and transitions. For a substantial number of our students, the second semester of 1999/2000 was not simply at extension of the first.

Significant Differences in Curriculum and Pedagogy

We use the term, mathematical discontinuities, to designate students' experiences of marked differences in mathematical expectations between their previous and current classrooms. We operationalize that term to be the differences that students report as they describe their current mathematics course. Sometimes in our interviews we have framed an explicit contrast to their past course; sometimes we have suggested that contrast implicitly (e.g., "what would you tell a graduating middle school [high school] student about your math class?") We address this issue in our first interview (early in the Fall semester) and we return to it in other interviews during the year, including our final interview in Spring 2000.

We present a preliminary sketch of what we have heard from students against the framework of curricular differences we identified in Figure 1. Then we identify some additional issues the students have mentioned that were not explicitly represented in Figure 1. Our identification and discussion of these differences is quite partial and incomplete as our analysis is on-going.

• Fundamental objects of study. We have no instances at all of students reporting a shift in the objects of study in their mathematics classes. It is not clear that the issues of unknowns vs. true variables and equations vs. functions are visible and/or important to them. We need to look more carefully at the "taught" curriculum to see if and when there were differences there to see. It may turn out that these distinctions are visible mainly to curriculum developers and mathematics education researchers.

• Typical problems. However, students frequently reported noticing changes in the character of problems. High school students at Site A (CMP graduates) often reported that they had usually worked on "story problems," while they are now working "book problems." ("Book problems" was just one of the many ways more "traditional" numerical or symbolic problems were described.) High school students at Site B (coming into Core-Plus) also reported that their problems now more frequently involved "real life" situations. Students at U-M frequently reported that their Harvard Consortium problems had more parts than they were used to. They also quickly became adept at distinguishing different kinds of problems based on their relative difficulty. Individual homework problems were judged to be the easiest; group homework problems the hardest.

• Typical solutions. At Site A, many students noticed changes in the character of problem solving in their classes. Where before, many students were centrally involved in generating solutions (often working in small groups) and different student methods were discussed, solution methods now flowed more directly from teachers and typically only one method was given. At U-M, many students reported that detailed verbal explanations were more highly valued than they expected. Giving "good" explanations required them to understand the content in ways that were qualitatively different than simply knowing how to solve the problem.

• Role of practice. Practice in mathematics entails two elements: (1) an explicit solution method and (2) a sequence of similar problems that can be solved by that method. Some students at both Site B and U-M have reported a shift from a long series of closely related homework problems to a much shorter list that may be related but do not submit to any single method of solution. We have found that different students express different preferences on this issue. Some prefer the traditional practice format because they say it is "easier;" others feel they do not learn more from practice once they have learned the method.

• Technology. Some students at Site B have noticed the time and attention that their teachers devote to learning to use graphing calculators (TI-83s). At the other sites, however, this has not been reported as an important dimension of difference. In part, this may be due to the fact that calculators (with and without graphing capabilities) are usually "around," whether their use in particular ways is actively supported (or not).

• Typical lessons. Somewhat surprisingly, students have not reported major differences in the ways their past and current teachers structure typical lessons. Discussion of prior homework, presentation of some new content, and time for individual or group problem solving appear to be standard lesson elements in many different classrooms.

Overall, the dimensions of Figure 1 have proven useful for structuring some of what students tell us about important differences. But other issues have not fallen easily into that framework. For example, at both universities, students frequently reported that they had to adjust to new, more distant, and/or less positive and productive relationships with their instructors than they had in high school. Also, some important differences may be site-specific. The Harvard Consortium materials in use at U-M mandates group homework assignments with challenging problems. Finding ways to negotiate this assignment with three students who are initially total strangers was a challenge for many students, whether they came to appreciate the assignments or not.

Discussion

[under construction: please accept our apologies]

Won't the problem, if it exists widely, go away naturally?

One objection to this research could be that the problem will go away naturally in a relatively short period of time. Curricular implementation, it may be argued, just takes time. In a few years, the curricular "spottiness" that underlies students' transitions will wane, as school districts (if not the country as a whole) trend toward much greater systemacity and coherence in their mathematics curricula and teaching. The problem of mathematical transitions will, to quote Marx, simply "wither away." While we cannot discount this possibility within another decade's time, we also see reasons to be less optimistic. Very different assumptions about the nature of mathematics, how it is best taught and learned, and who can learn it well underlie the curricular decisions that led to "spottiness" in the first place. In our first year of work, we have not seen the conditions, within our districts and sites, for any easy resolution of these differences. Indeed even the time for genuine dialogue, much less the necessary good will and openness, is hard to find. Without focused attention to documenting students' experiences and making sense of them with mathematics educators, we see little hope for easy resolution in the short-term.

<u>Our goals are practical as well as scholarly</u>. We hope to provide useful analyses to mathematics educators at each site to facilitate their efforts to identify and smooth transitional "bumps." For example, at Site A mathematics teachers at the middle and high school both see substantial differences in curricular approach in each building but do not agree on the character of desirable curricula or how best to engage students in learning. There is some evidence that our results may help them find more common ground than they currently claim.

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